Endogenous Maximum RPM, Recommended Retail Prices and the Role of Buyer Power

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Abstract

This paper investigates whether, and under which conditions, in a vertical structure it would be preferable to guarantee the retailer the ability of setting retail prices—by permitting only a Recommended Retail Price (RRP)—or rather, to give it to the manufacturer, by allowing a Maximum Resale Price Maintenance (RPM). This issue has been crucial in important antitrust disputes on vertical maximum price fixing, but it has not been studied theoretically yet. This paper aims at filling this gap, looking also at the effect of buyer power on the equilibrium vertical price restriction. It is shown that the manufacturer can offer the retailer a high unit discount to induce her to accept a Maximum RPM. RRP can be an equilibrium solution either when buyer power is very low or very high. In the intermediate case when buyer and seller power are balanced an equilibrium maximum RPM endogenously occurs. The latter would be the best situation for society. The Galbraith conjecture is then at work here. We find no reason, unlike the current attitude of antitrust authorities, to prefer RRP to maximum RPM.

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1 Introduction

In a few industries a manufacturer recommends a retail price to his retailers and, when selling the good, gives them a pre–determined unit discount computed as a deduction on that price. The use of a Recommended (Suggested) Retail Price (RRP) is often observed in markets like, for example, gasoline, cars, medicines, books, magazines, clothing and beer. In some cases this may become a Maximum Resale Price Maintenance (RPM),[1] because retailers, having received a satisfactory unit discount, agree to leave the price set by the manufacturer unchanged. The amount of the retailers' discount is however variable from industry to industry, and depends on their buyer power in dealing with manufacturers.

Vertical price restrictions have traditionally been considered harmful for society by antitrust authorities and, consequently, forbidden. In recent years, however, the US Supreme Court has overruled the *per se* illegality of Maximum RPM in the *State Oil v. Khan et al.* [1997] case. The European Commission has shared this view in 1999 (Rule n. 2790, art. 4(a)). As a result, Maximum RPM is now permitted both in the EU and in the US.[2] Other weaker forms of price restrictions, like RRP, are also permitted. Hence, both Maximum RPM and RRP are now available options.[3]

Some interesting yet unanswered questions immediately arise: which of

[1]This may well be desirable for the manufacturer since it would eliminate double marginalization and increase his demand.

[2]Minimum and plain RPM are instead still forbidden in both areas.

[3]Even under the most severe legislation of Canada and the UK, in which RPM has been formally banned, Maximum RPM and RRP are currently permitted. See Mathewson–Winter [1998] for more details.

these two available vertical price restrictions prevail in equilibrium and under what conditions (for instance, in terms of buyer power)? Which of them should be preferred from a welfare point of view? In other words, how important is to insure the retailer the freedom to set the retail price?[4]

Bork [1978] and Posner [1976] argued that vertical restraints, inclusive of price restrictions, are pro–competitive, because the aim of the manufacturer in managing them is to increase sales, which also make consumers happier. But when buyers (i.e. retailers) do have market power—for instance because it is difficult to replace them[5]—this argument does not always apply, given that it can be costly to convince them to accept a vertical restraint which may well make their profit lower. It then becomes important to investigate how the desire of the manufacturer to affect the retail price interacts with the retailer's buyer power.

Our aim is precisely to look at this question. We see two channels for the retailer to exert her buyer power: (1) the determination of the unit discount

[4]It is to be noted, however, that according to the prevailing US antitrust doctrine, which follows the conclusions reached in the *General Electric* [1926] case, it is crucial—to assess *who* should set the retail price—to understand the true nature of the retailer's activity: when the retailer is a mere agent of the manufacturer, she is not entitled to set the retail price and the manufacturer should. This occurs when the retailer is not the owner of the good for sale and does not bear any entrepreneurial risk (for instance, when she does not hold an inventory and/or an investment risk). In this case vertical price restrictions are not an issue. On the contrary they become a hot issue when the retailer acts as a firm.

[5]Another cause for buyer power can be the adoption of non–price vertical restrictions, e.g. exclusive dealing and/or exclusive territories used, for instance, to give the right incentive to retailers to provide enough sales promotions.

and (2) the ability of affecting the retail price (i.e. RRP *vs* RPM). When the second channel is considered, the manufacturer might indeed use the retailer's discount as an instrument to buy her willingness to control the retail price, but this could be costly.

The traditional literature on RPM has not dealt with these questions: rather, it has concentrated its efforts in investigating whether RPM, alone or jointly with other vertical restraints (e.g. franchise fees, exclusive territories, quantity fixing), may replicate the vertical integration solution, thus correcting a variety of horizontal and/or vertical externalities, such as double marginalization, excessive downstream (price and nonprice) competition and Telser's [1960] free riding argument on retail services.[6] Other contributions have highlighted the *cartel hypothesis*, i.e. RPM might be a device to implement collusion either between manufacturers (Jullien–Rey [2003]) or among retailers (Shaffer [1991], Wang [2004]).

Literature has focused on Minimum RPM, the most controversial case, devoting less attention to Maximum RPM. Moreover, it has seldom taken into consideration manufacturers' recommended retail prices (Rosenkranz [2003] is one of the very few papers)[7] and has never brought into the picture the

[6] In this line of research we can mention Mathewson–Winter [1983, 1984], Dixit [1983], Perry–Groff [1985], Perry–Porter [1990], and Bolton–Bonanno [1988]. The common point they make is that RPM alone is not able to achieve the vertical integration solution (i.e. the first best) but the latter can be obtained if RPM is adopted *together with* other vertical restraints. Rey–Tirole [1986] have, among others, looked at the vertical control problem from a different perspective, a principal–agent approach, emerging when uncertainty and asymmetric information are considered.

[7] She considers, unlike our perspective, a model where the manufacturer advertises a recommended price and consumers react to it by reducing (increasing) their willingness to

role of buyer power[8] in choosing a vertical price restriction (the contribution of Klein–Murphy [1988][9] and Perry–Besanko [1991] are partial exceptions). Retailers have usually been regarded as agents with no buyer power (e.g. Rey–Tirole [1986], Tirole [1988]), on the grounds that they can be easily replaced by the manufacturers, while this assumption is clearly violated in some important industries (e.g. food, gasoline, cars and books), where the retailer instead controls important tangible or intangible assets in the vertical channel.

We show that a retailer's buyer power does matter in determining the equilibrium vertical price restriction and, consequently, it has important effects on consumer surplus, channel profit and welfare. An endogenous RPM emerges only when retailer's buyer power is well matched with the manufacturer's seller power. When instead the parties' bargaining power is highly unbalanced, RRP is the equilibrium solution. It may seem counter–intuitive that the retailer chooses to give away the retail price control when she has

pay if the retailer charges a price higher (lower) than the recommended one. We believe that this situation can be applied to few markets, since consumers are rarely informed about the manufacturer's recommended price.

[8]It exists an extensive recent literature focusing on buyer power in vertical relations between firms (e.g. von Ungern–Sternberg [1996], Dobson–Waterson [1997] and Inderst–Wey [2006], but not on its effects on the vertical restraints choice.

[9]Klein–Murphy consider that retailer's buyer power may be desirable for the manufacturer to eliminate a moral hazard problem. When consumers cannot recognize, before the purchase, whether the standards of quality recommended by the manufacturer have been followed by an individual retailer, the latter has an incentive to shirk and so enjoys a higher margin by cutting her costs. Klein–Murphy show that giving more market power to retailers and granting them enough remuneration avoids retailer's shirking, given that they fear to lose, through contract termination, a remunerating business.

a (relatively) high buyer power, and instead to keep control over it when her buyer power is small. But the explanation is that the retailer uses her buyer power to increase the predetermined unit discount. When the latter is sufficiently high, she no longer needs to control the retail price. However, a very high buyer power reduces the manufacturer's incentive to offer a RPM contract, since it is too costly for him to induce the retailer to leave the price control.

Looking at welfare, we show that, in equilibrium, RRP is dominated by Maximum RPM. There is then no reason to prefer RRP to Maximum RPM, as sometimes suggested by antitrust authorities. When RRP prevails (i.e. when buyer power is either low or high), it gives rise to a welfare loss due to both double marginalization and a positive discount. When Maximum RPM prevails, double marginalization is eliminated, but the distortion due to the unit discount is still present. However the latter negative effect is dominated by the former positive effect, and so welfare is higher (even if decreasing with buyer power). Galbraith's countervailing power hypothesis is thus at work in this model.

Our study focuses on a second best analysis in that, although vertical integration and/or wholesale nonlinear pricing would be more efficient than linear pricing and unit discounts, they are not adopted. This can be due to high fixed costs for vertical integration (which then becomes unattractive), and to moral hazard problems in splitting the channel profit through wholesale nonlinear pricing.

The paper proceeds as follows. In Section 2 we briefly recall the antitrust controversy on Maximum RPM, by looking at the two most important cases.

In Section 3 we present our model based on the common features shown in the studied antitrust cases. In Section 4 we study the retail price determination. In Section 5 we analyze the setting of the recommended retail price by the manufacturer. In Section 6 we characterize the optimal vertical price restriction under various degrees of retailer's buyer power, also looking at its effect on profits and welfare. Concluding comments are included in Section 7. Analytical details provided in the Appendix end the paper.

2 The antitrust controversy on Maximum RPM

The American antitrust authorities have long held *per se* illegality on both Maximum and Minimum RPM. This doctrine started from *Dr. Miles v. J. Park & Sons Co.* [1911] and has been applied continuously along decades in several cases. The last famous case where this doctrine has been applied is *Albrecht v. Herald Co.* [1968], concerning the newspaper home–delivery industry.[10] In the late 90's however, the US Supreme Court has overruled *per se* illegality for Maximum RPM in *State Oil v. Khan et al.* [1997], a gasoline retailing case.[11] The Court stated that, since it was not sure

[10] The publisher of a famous daily newspaper in the St.Louis area signed a contract with independent newspaper carriers for home delivering, granting them monopoly in distribution within a given area. However the publisher set a maximum retail price, and gave to carriers a predetermined unit discount computed as a deduction from that maximum retail price. One of these carriers—Albrecht—did not meet the maximum price, and, afterward, sued the publisher for violation of Section 1 of the Sherman Act.

[11] An independent gasoline retailer—*Khan et al.*—entered into a lease and supply agreement to operate a gasoline station and a convenience store owned by *State Oil*. The agreement provided that the retailer would exclusively obtain his supply of gasoline from the oil company at a price equal to a suggested retail price set by *State Oil*, less a margin

anymore that Maximum RPM would have reduced welfare, the traditional *per se* illegality of *Dr. Miles* should be overruled. This change in the American antitrust authorities' view makes a theoretical analysis of Maximum RPM very interesting. The bulk of the question is the following: who should set, from a welfare point of view, the retail price when both upstream and downstream firms have market power? Is society better off under a Maximum RPM or under a RRP?

This paper deals with these questions in a simple theoretical model, in which we will nest the three main common features of the two antitrust cases before mentioned. They are: (1) a manufacturer sells a good to a retailer who is vertically separated and market–powered; (2) the manufacturer can set a maximum RPM, which is binding for the retailer only if she has accepted it; (3) the retailer is compensated on the basis of a predetermined unit discount computed as a deduction from the given maximum RPM (or from the given RRP).[12]

of 3.25 cents per gallon. The agreement did not specify the price the retailer could charge for gasoline. If, however, he charged more than the oil company's suggested retail price, he had to rebate to the oil company the entire amount gained. This implies that a binding Maximum RPM has been set by the oil company. The legal controversy started as soos as the retailer fell behind in lease payments as a result of the manufacturer's retail price requirement.

[12]These features are common to many other cases and thus represent elements of a rather standard situation.

3 The model

Let us then consider a successive monopoly where a manufacturer sells his product to a single retailer, with $y(p)$ representing the downward sloping demand function by final consumers ($y' < 0$). For the sake of simplicity, let us assume that each unit of the good bought by the retailer is sold to consumers and that the retailer does not incur any distribution costs. Her marginal cost is thus equal to the wholesale price of the good. The manufacturer produces with constant unit cost, which is, without loss of generality, normalized to zero.

We define $\overline{p}$ as the recommended retail price (RRP) set by the manufacturer and s as the unit discount granted to the retailer and deducted on the recommended price. Hence the wholesale price can be defined as $w(\overline{p}, s) = \overline{p} - s$. The retail price is p, so the retailer's profit margin is $p - w(\overline{p}, s) = p - (\overline{p} - s)$.[13] If $p = \overline{p}$, the recommended retail price is confirmed by the retailer and her profit margin is only equal to s. If instead $p > \overline{p}$, the retailer's profit margin rises and overpricing takes place. So the retailer has two ways to increase her profit margin: (1) overpricing, (2) an increase in s. If instead $p < \overline{p}$, she charges a price lower than the recommended one (underpricing), and her profit margin shrinks.[14] We assume that

[13] The presence of a strictly positive unit discount s therefore introduces a wedge between the recommended retail price $\overline{p}$ and the wholesale price w. Indeed, when $s = 0$, $\overline{p}$ represents both the retail price recommended by the manufacturer and the wholesale price, but this is no longer true when $s > 0$. It follows that in presence of $s > 0$ and if the buyer has the ability to affect it, the wholesale price is not decided by the manufacturer alone, but it is partly determined by the retailer, giving rise to a case of buyer power.

[14] However, the retailer's profit may increase if the reduction in the profit margin due to

the retailer can exert her negotiation power through both s and p, i.e. she has some bargaining power in determining s and she may refuse to charge the recommended price $\overline{p}$ proposed by the manufacturer.

In order to model the two cases where a Maximum RPM is respectively present or absent, we proceed as follows. There are two contracts: 1) a RRP contract (labeled RRP), in which the retailer remains completely free to choose p, even above $\overline{p}$; 2) a Maximum RPM contract (labeled RPM) such that the retail price must not be higher than the recommended one (i.e. $p \leq \overline{p}$).

Clearly, the manufacturer can also choose not to offer the RPM contract if this is not profitable for him. Convincing the retailer to accept it can be costly, and, hence, he could prefer not to offer it, and remain in the RRP regime.[15] We assume that both the manufacturer's decision to offer either RRP or RPM and the retailer's eventual decision to accept RPM take place knowing s. The impact of different games with alternative timing patterns on the equilibrium vertical contract will be discussed later in Sub–Section 6.4.

The magnitude of s depends crucially on buyer power. We define β, with $0 \leq \beta \leq 1$, as the degree of buyer power in the determination of s. If $\beta = 0$, the retailer has no buyer power on s (but she can still refuse the RPM contract); if $\beta = 1$, s is determined by the buyer.[16]

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underpricing is more than compensated by the increase in the market demand.

[15] As we will see later, it can be shown that the RRP regime is equivalent to setting a uniform wholesale price, so that there is no loss in generality in restricting the manufacturer's choice to only two contracts.

[16] As already explained fixed fees and vertical integration are not available.

The whole situation can be formally described as a perfect and complete information four–stage game, with the following timing:

- at $t = 1$ (the *discount* stage) the retailer's unit discount s is set as a solution of an asymmetric Nash bargaining problem where buyer power β (and also seller power $1 - \beta$) plays a crucial role;

- at $t = 2$ (the *retail price regime* stage) the manufacturer, knowing s, decides whether to offer a RPM contract or not, and the retailer chooses whether to accept it or not. Hence in this stage the two firms choose between RRP and RPM. The latter prevails only when offered by the manufacturer and accepted by the retailer;

- at $t = 3$ (the *recommended retail price* stage), the manufacturer, knowing s and the prevailing retail price regime (i.e. RRP or RPM), chooses the recommended price $\overline{p}$;

- at $t = 4$ (the *retail price* stage), the retailer sets the retail price p, with the restriction that $p \leq \overline{p}$ if the RPM contract prevails, and with no constraints under RRP.

We look for the subgame perfect equilibria of this game, applying, as usual, backward induction. For this reason we start the analysis, in the next section, from the last stage.

4 Retail Price Determination

The prevailing retail price regime (determined at $t = 2$) clearly affects what happens in the last stage, where the retailer sets p. We need then to distinguish between the two possible cases: (a) the retailer chooses p under RRP (i.e. she is unconstrained on the retail price), (b) the retailer selects p under RPM (i.e. $p \leq \overline{p}$). The former case is considered first.

4.1 RRP

If the retailer is unconstrained on the retail price, her profit can be written as follows:

$$\pi_R^{RRP} = [p - (\overline{p} - s)]y(p) \tag{1}$$

It is clear from (1) that underpricing is subject to a lower bound provided by the non–negativity constraint on profits, $\pi_R \geq 0$, which implies that $p \geq \overline{p} - s$. The retailer chooses p at $t = 4$ in order to maximize (1). The first order condition of this profit maximization can be written as follows:

$$\frac{d\pi_R^{RRP}}{dp} = y(p) + (p - \overline{p} + s)y'(p) = 0 \tag{2}$$

The following Lemma is easily proved:

Lemma 1 *Under RRP, the implicit function* $p = \phi(\overline{p}, s)$ *holds, with* $\frac{\partial\phi(\overline{p},s)}{\partial\overline{p}} = -\frac{\partial\phi(\overline{p},s)}{\partial s} > 0$ *and* $\frac{\partial^2\phi}{\partial\overline{p}\partial s} = \frac{\partial^2\phi}{\partial s\partial\overline{p}}$.

Proof: See the Appendix.

4.2 RPM

If the retailer has accepted the Maximum RPM contract at $t = 2$, her maximization problem is the following:

$$\begin{aligned} &Max_p \quad \pi_R^{RPM} = [p - (\overline{p} - s)]y(p) \\ &s.t. \qquad p \leq \overline{p} \end{aligned} \tag{3}$$

In order to see whether the constraint is binding or not, given the assumed concavity of the profit functions, it is sufficient to compute the first order derivative of π_R^{RPM} relative to p when $p = \overline{p}$. It yields:

$$\left.\frac{d\pi_R^{RPM}}{dp}\right|_{p=\overline{p}} = y'(\overline{p})\left[\frac{y(\overline{p})}{y'(\overline{p})} + s\right] \tag{4}$$

When (4) is not negative, we are sure that the constraint $p \leq \overline{p}$ is binding. Under these circumstances the profit–maximizing retailer selects $p = \overline{p}$. The condition under which this occurs is easily computed from (4) and expressed in the following Lemma.

Lemma 2 *Under RPM, $p = \overline{p}$ if the following condition holds:*

$$s \leq -\frac{y(\overline{p})}{y'(\overline{p})} \tag{5}$$

Lemma 2 shows that only if the unit discount is sufficiently low, RPM leads to a retail price equal to the ceiling determined by the manufacturer and accepted by the retailer. This implies that, under the RPM regime, the manufacturer, when computing the recommended price, has to take into account that the level of s has an impact on the retailer's behavior: she may underprice or confirm the recommended price. When the retailer underprices $\overline{p}$, p is derived from (2), i.e. $p = \phi(\overline{p}, s)$. Hence the retail price under RPM is the following:

$$p^{RPM} = \begin{cases} \overline{p}^{RPM} & \text{if (5) holds} \\ \phi(\overline{p}^{RPM}, s) < \overline{p}^{RPM} & \text{otherwise} \end{cases} \tag{6}$$

5 The retail price recommended by the manufacturer

Having solved the last subgame, we are now in a position to look at what happens at $t = 3$, where the manufacturer sets the recommended retail price. He will set $\overline{p}$ anticipating the retailer's decision concerning p, which, in turn, depends upon the prevailing retail price regime. Indeed we know from Lemma 1, that under RRP $p^{RRP} = \phi(\overline{p}^{RRP}, s)$; if instead RPM is adopted, the retail price is defined by (6). Hence we have again to consider the following two cases.

5.1 RRP

In this case the manufacturer's profit is given by $\pi_M^{RRP} = (\overline{p} - s)y[\phi(\overline{p}, s)]$. He maximizes π_M^{RRP} by choosing $\overline{p}$. The first order condition is as follows:

$$\frac{d\pi_M^{RRP}}{d\overline{p}} = y[\phi(\overline{p}, s)] + (\overline{p} - s)y'[\phi(\overline{p}, s)]\frac{\partial \phi}{\partial \overline{p}} = 0 \tag{7}$$

The implicit function $\overline{p}^{RRP} = \varrho(s)$ yields the recommended price. Moreover, after some algebraic manipulations (see the proof of Lemma 3 in Appendix) we show that it must be:

$$\frac{d\overline{p}^{RRP}}{ds} = 1 \tag{8}$$

so that $\overline{p} = \kappa + s$, where κ is a strictly positive constant. Hence p^{RRP} is independent of s, given that a variation in s has no effect on $w(\overline{p}, s)$. The following Lemma thus holds:

Lemma 3 *Under RRP, the retail price is independent of s. Hence consumer surplus is unaffected by s.*

Proof: See the Appendix.

Lemma 3 points out that if the retailer asks for a higher s, the manufacturer compensates this loss by increasing $\overline{p}$ by the same amount, in such a way as to keep the wholesale price constant. This in turn leaves the retail price unchanged, and, hence, consumer surplus as well.

The effect of s on welfare under RRP is now considered and shown in the following Proposition.

Proposition 1 *Under RRP, the unit discount s does not affect firms' profits, consumer surplus and social welfare. Only the recommended retail price increases with s.*

Proof: See the Appendix.

A corollary of Proposition 1 is that, under RRP, buyer power, acting through s, has no relevant effect, since welfare is independent of s.[17]

5.2 RPM

Let us now analyze the case where the retailer has accepted the RPM contract. Given that p^{RPM} is defined by (6), two cases have to be considered.

[17]Hence, the retailer's decision on p (i.e. overpricing, underpricing, or confirming $\overline{p}$) is, under these conditions, irrelevant, since it does not affect any economic variable.

a) The constraint $p^{RPM} \leq \overline{p}^{RPM}$ is binding

Since $p^{RPM} = \overline{p}^{RPM}$, market demand is no longer function of $p = \phi(\overline{p}, s)$ but directly of the recommended price. The manufacturer maximizes the following profit: $\pi_M^{RPM} = (\overline{p} - s)y(\overline{p})$. The first order condition to have a maximum is:

$$\frac{d\pi_M^{RPM}}{d\overline{p}} = y(\overline{p}) + (\overline{p} - s)y'(\overline{p}) = 0 \tag{9}$$

The following Lemma can be shown to hold.

Lemma 4 *Under RPM, when the constraint $p^{RPM} \leq \overline{p}^{RPM}$ is binding, the recommended price is defined by the implicit function $\overline{p}^{RPM} = \varphi(s)$, with $0 < \frac{d\overline{p}}{ds} < 1$.*

Proof: See the Appendix.

Lemma 4 shows that $\overline{p}^{RPM}$ increases with s, but not with a unitary change ratio, as it is under RRP.

b) The constraint $p^{RPM} \leq \overline{p}^{RPM}$ is not binding

The retail price in this case is defined by $p = \phi(\overline{p}, s) < \overline{p}^{RPM}$. The manufacturer's profit is, again, $\pi_M^{RPM} = (\overline{p} - s)y[\phi(\overline{p}, s)]$ and so (7) holds. The implicit function $\overline{p}^{RRP} = \varrho(s)$ again holds. In other words, if the retailer has accepted the RPM contract but the price restriction is not binding, it is like being in the RRP regime. Under these circumstances the two retail price regimes coincide. Hence Lemma 1, Lemma 3 and Proposition 1 apply.

Having studied the two possible cases arising under RPM, we can specify the equilibrium recommended price functions prevailing at $t = 3$ as follows:

$$\overline{p}^{RPM} = \begin{cases} \varphi(s) & \text{if (5) holds} \\ \varrho(s) & \text{otherwise} \end{cases} \tag{10}$$

We can now state the following Proposition concerning the impact of s on the endogeneous variables in the RPM regime.

Proposition 2 *Under RPM, if the price constraint is binding, then both industry profit and consumer surplus decrease as s increases, so that an increase in s is welfare reducing. If instead the price constraint is not binding, then RPM coincides with RRP and Proposition 1 holds.*

Proof: See the Appendix.

This result points out that under RPM, when $p = \overline{p}$, an increase in s would give to the retailer more profit, but since both the manufacturer and the consumers would be hurt by the consequent retail price increase, the overall effect on welfare would certainly be negative.[18] The retailer's unit discount acts as a new distortion to be traded off with the elimination of double marginalization.

It is important to remark that the level of s determines whether the retailer, under RPM, will confirm $\overline{p}^{RPM}$ or not (see Lemma 2). Indeed, condition (5) will be met only when s is small enough.[19]

[18] Under RRP instead, recall that s is neutral on welfare.

[19] Condition (5) is endogenous, since, as shown in (10), $\overline{p}^{RPM}$ depends upon it. However, $\overline{p}^{RPM} = \varphi(s)$ is computed under the assumption that $p^{RPM} = \overline{p}^{RPM}$, so that $y(\overline{p}^{RPM})$. If the manufacturer anticipates that the retailer will underprice the recommended price, his

5.3 Comparing the two retail price regimes

Having solved the last two subgames under the two possible retail price regimes, we can compare prices, channel profit, consumer surplus and welfare under RRP and RPM. The results are shown in the following Proposition:[20]

Proposition 3 *Comparing RPM (with a binding price constraint) with RRP, consumer surplus, channel profit and welfare are lower (higher) under RPM than under RRP if s is sufficiently high (low).*

Proof: See the Appendix.

Proposition 3 shows that, on one hand, under RPM (with $p = \overline{p}$) the manufacturer eliminates the retailer's markup and this makes the vertical channel more efficient. But, on the other hand, a higher s makes the recommended price's increase, even though less than s (see Lemma 4). This brings about a reduction in the manufacturer's profit. The retailer gets a higher profit, but not enough to compensate the reduction in the manufacturer's profit. Industry profit then decreases under RPM with a high s. Also the retail price increases with s, due to the recommended price increase. The consumer surplus thus decreases and welfare as well. When s is sufficiently high, RPM is hence undesirable from a social point of view. The price increase due to a high s can well prevail on the social gain due to the elimination of double marginalization.

rational behavior is to charge $\overline{p}^{RPM} = \varrho(s)$, since $y(p)$. Hence the relevant recommended price for satisfying condition (5) is $\overline{p}^{RPM} = \varphi(s)$, given that it gives rise to $p^{RPM} = \overline{p}^{RPM}$.

[20]Obviously there is no point in comparing the two price regimes when, under RPM, the retailer does not confirm $\overline{p}$, because in this case RPM collapses to RRP.

Since all depends on the level of s, it is now important to determine its equilibrium level, which, in turn, is function of the parties' bargaining power. We will do it in the next Section.

6 Equilibrium vertical price restrictions and buyer power

We now turn to the first two stages of the model, where s and the retail price regime have to be determined. We want to examine the effect of the buyer power expressed by β on the equilibrium vertical price restriction. As already mentioned, at $t = 1$, s is determined as the solution of an asymmetric Nash bargaining problem. At $t = 2$, given s, the manufacturer decides whether to offer a RPM contract or not, and the retailer chooses whether to accept it or not. We want to find the subgame perfect equilibria. To achieve this goal, however, we need to obtain closed form solutions. To this purpose we now assume a linear specification for the final demand: $y = a - p$. Under this assumption, the subgames at stages 3 and 4 yield the outcomes shown in Table 1.[21] The critical level of s, defined as s', that makes the price constraint binding in the RPM regime, is obtained by substituting in (5) $\overline{p}^{RPM} = \frac{a+s}{2}$ (with $y = a - p$ and $y' = -1$). We get the following inequality: $s \leq \frac{a-s}{2}$. Solving it for s we obtain the maximal s satisfying it: $s' = \frac{a}{3}$. Hence, we have that $\overline{p}^{RPM} = \frac{a+s}{2}$ if $s \leq \frac{a}{3}$ while $\overline{p}^{RPM} = \frac{a}{2} + s$ if $s > \frac{a}{3}$.

To highlight the role of buyer power, let us now focus on the polar cases: (1) the case where there is no buyer power (i.e. $\beta = 0$ and no chance of refusing the RPM regime) and (2) the case of maximum buyer power (i.e.

[21] It is straightforward to show that, under RRP, underpricing will occur only if $s > \frac{a}{4}$; for $s < \frac{a}{4}$ we will have overpricing, while for $s = \frac{a}{4}$ we will have an endogenous RPM.

Vertical price restriction	Outcomes							
	p	$\overline{p}$	y	π_R	π_M	CS	Π	W
RRP	$\frac{3a}{4}$	$\frac{a}{2}+s$	$\frac{a}{4}$	$\frac{a^2}{16}$	$\frac{a^2}{8}$	$\frac{a^2}{32}$	$\frac{3a^2}{16}$	$\frac{7a^2}{32}$
RPM ($p^{RPM}=\overline{p}^{RPM}$)	$\frac{a+s}{2}$	$\frac{a+s}{2}$	$\frac{a-s}{2}$	$\frac{s(a-s)}{2}$	$\frac{(a-s)^2}{4}$	$\frac{(a-s)^2}{8}$	$\frac{a^2-s^2}{4}$	$\frac{(3a+s)(a-s)}{8}$
RPM ($p^{RPM}<\overline{p}^{RPM}$)	$\frac{3a}{4}$	$\frac{a}{2}+s$	$\frac{a}{4}$	$\frac{a^2}{16}$	$\frac{a^2}{8}$	$\frac{a^2}{32}$	$\frac{3a^2}{16}$	$\frac{7a^2}{32}$

Table 1: Outcomes in Stages 3–4 with linear demand

$\beta = 1$ and ability of refusing RPM).

6.1 The polar cases: no buyer power and maximum buyer power

If buyer power is zero, then at $t = 2$ the manufacturer imposes the RPM contract and gets $\pi_M^{RPM} = \frac{(a-s)^2}{4}$, since $p^{RPM} = \overline{p}^{RPM} = \frac{a+s}{2}$. At $t = 1$ the manufacturer sets s; being $\frac{d\pi_M^{RPM}}{ds} < 0$ we will have $s^* = 0$. This implies that $w = \overline{p} = p$, $p^{RPM} = \frac{a}{2} = y$ and $\pi_R = 0$, while $\pi_M = \frac{a^2}{4} = \pi^{VI}$ (VI stands for vertical integration). Channel profit coincides with manufacturer's profit and $W = \frac{3}{8}a^2 = W^{VI}$. Under these circumstances, RPM replicates vertical integration and achieves the first best solution.

The analysis is less straightforward when buyer power is maximum. In this case at $t = 2$ the retailer may refuse RPM. If she accepts it, her profit is:

$$\pi_R^{RPM} = \begin{cases} \frac{s(a-s)}{2} & \text{if} \quad s \leq \frac{a}{3} \\ \frac{a^2}{16} & \text{otherwise} \end{cases}$$

If $s > \frac{a}{3}$ RPM collapses to RRP. If $s \leq \frac{a}{3}$ the retailer will accept RPM if $\pi_R^{RPM} = \frac{s(a-s)}{2} \geq \frac{a^2}{16} = \pi_R^{RRP}$. This inequality if satisfied for $s_1 \leq s \leq s_2$ where:

$$s_1 = \frac{a}{2}\left(1 - \frac{\sqrt{2}}{2}\right) > 0, s_2 = \frac{a}{2}\left(1 + \frac{\sqrt{2}}{2}\right) > 0, \quad \text{with} \quad s_1 < \frac{a}{3} < s_2. \quad (11)$$

Hence the retailer will accept RPM only if $s_1 \leq s \leq \frac{a}{3}$. We have now to see whether it is profitable for the manufacturer to offer the RPM contract or not. When RPM collapses to RRP the manufacturer will not offer it in the first place. When $s \leq \frac{a}{3}$ the crucial condition for the manufacturer to offer RPM is: $\pi_M^{RPM} = \frac{(a-s)^2}{4} \geq \frac{a^2}{8} = \pi_M^{RRP}$. This inequality if satisfied for $s \leq s_3$ and $s \geq s_4$ where:

$$s_3 = a\left(1 - \frac{\sqrt{2}}{2}\right) > 0, s_4 = a\left(1 + \frac{\sqrt{2}}{2}\right) > 0, \quad \text{with} \quad s_1 < s_3 < \frac{a}{3} < a < s_4 \quad (12)$$

From (12) we know that the manufacturer will offer the RPM contract only if $s \leq s_3$.[22] Let us now look for the solution at $t = 1$, where the retailer chooses s in order to maximize her profit. It is easy to see that $s^* = s_3$ and so RPM prevails, with $p^{RPM} = \overline{p}^{RPM} = \frac{a}{4}\left(4 - \sqrt{2}\right)$, $\pi_R^{RPM} = \frac{a^2}{4}\left(\sqrt{2} - 1\right)$, $\pi_M^{RPM} = \frac{a^2}{8}$, $\Pi = \frac{a^2}{8}\left(2\sqrt{2} - 1\right)$, $CS^{RPM} = \frac{a^2}{16}$ and $W^{RPM} = \frac{a^2}{16}\left(4\sqrt{2} - 1\right)$. When buyer power is maximum RPM prevails, but the first best is not achieved. There is a distortion in prices due to the high buyer power.

6.2 Asymmetric Nash Bargaining on s

Let us now generalize buyer power by allowing β to assume any value between the two polar cases already studied ($0 \leq \beta \leq 1$). We setup this problem as a standard asymmetric Nash bargaining. At $t = 2$ the retailer has still the

[22] The solution $s \geq s_4$ is unfeasible.

ability to refuse RPM, while the manufacturer will consider whether to offer the RPM contract is profitable or not. Hence equations (11)–(12) hold. At $t = 1$ the manufacturer and the retailer solve the following problem:

$$\underset{\{s\}}{\text{MAX}} \qquad \left(\pi_R - \pi_R^d\right)^{\beta} \left(\pi_M - \pi_M^d\right)^{1-\beta} \tag{13}$$

where π_j^d $(j = R, M)$ is firm j's disagreement profit. If the two parties do not reach an agreement, no trade takes place and hence both firms end up with no profit: so we have $\pi_R^d = \pi_M^d = 0$. Hence (13) can be simplified to:

$$\underset{\{s\}}{\text{MAX}} \qquad \pi_R^{\beta} \pi_M^{1-\beta} \tag{14}$$

If the retailer rejects RPM at $t = 2$, RRP prevails and hence both profits are independent of s; under these circumstances, the bargaining over s is irrelevant. Firms stick to a profit of $\pi_R^{RRP} = \frac{a^2}{16}$ and of $\pi_M^{RRP} = \frac{a^2}{8}$. If manufacturer offers the RPM contract and the retailer instead accepts it, a non trivial bargaining problem applies and gives rise to the following maximization problem:

$$\underset{\{s\}}{MAX} \qquad \left(\pi_R^{RPM}\right)^{\beta} \left(\pi_M^{RPM}\right)^{1-\beta} = \left[\tfrac{s}{2}\left(a-s\right)\right]^{\beta} \left[\tfrac{(a-s)^2}{4}\right]^{1-\beta} \tag{15}$$

In this case s does affect both profits. The following first order condition can be easily obtained:

$$\begin{gathered} \beta \left[\tfrac{s}{2}\left(a-s\right)\right]^{\beta-1} \left[\left(\tfrac{1}{2}\right)\left(a-s\right) - \tfrac{s}{2}\right] \left[\tfrac{(a-s)^2}{4}\right]^{1-\beta} + \\ -\tfrac{1-\beta}{2} \left[\tfrac{s}{2}\left(a-s\right)\right]^{\beta} \left[\tfrac{(a-s)^2}{4}\right]^{-\beta} \left(a-s\right) = 0 \end{gathered} \tag{16}$$

Solving (16) for s we get

$$s^{RPM^*} = \frac{a\beta}{2} \tag{17}$$

which implies, since $0 \leq \beta \leq 1$, that $0 \leq s^{RPM^*} \leq \frac{a}{2}$. However, from (11) and (12), we know that RPM will be offered and accepted only if $s_1 \leq s \leq s_3$. Since s^{RPM^*} depends upon β, and given that $s_1 = \frac{a}{2}(1 - \frac{\sqrt{2}}{2})$, we have $s_1 = s^{RPM^*}$ when $\beta = \beta_1 = 1 - \frac{\sqrt{2}}{2}$, while $s^{RPM^*} = s_3$ when $\beta = \beta_3 = 2 - \sqrt{2}$. Hence the following Proposition holds.

Proposition 4 *When a manufacturer and a retailer bargain over s and the retailer has the ability to refuse a RPM contract, RRP prevails if buyer power is either small, i.e. $0 \leq \beta < \beta_1$ or large, i.e. $\beta_3 < \beta \leq 1$. When instead buyer power is moderate (i.e. $\beta_1 \leq \beta \leq \beta_3$), RPM arises.*

Proposition 4 points out that buyer power on s, measured by β, affects the nature of the vertical price restriction arising in equilibrium. If buyer power is small, the retailer gets a small s at $t = 1$. For this reason she refuses RPM, because under RRP she enjoys a higher profit by practicing overpricing. If instead buyer power is large, the retailer would accept RPM, because the remuneration obtained through s is satisfactory enough for her to accept losing control over p. She realizes that RPM would improve the channel efficiency and also that, through a high s, she could get a high portion of the channel surplus. The manufacturer, however, if β is large, will not offer RPM, since it is too costly for him to convince the retailer to lose control of the retail price. For this reason RPM will arise only when buyer power is moderate and similar to seller power, i.e. in case of Galbraith countervailing hypothesis.

β	Retail pricing regime	π_M	π_R
$0 \leq \beta < \beta_1$	RRP	$\frac{a^2}{8}$	$\frac{a^2}{16}$
$\beta_1 \leq \beta \leq \beta_3$	RPM	$\left[\frac{a^2}{32}\left(3+\sqrt{2}\right), \frac{a^2}{8}\right]$	$\left[\frac{a^2}{16}, \frac{a^2}{16}\left(\sqrt{2}-1\right)\right]$
$\beta_3 < \beta < 1$	RRP	$\frac{a^2}{8}$	$\frac{a^2}{16}$

Table 2: The effect of buyer power

Table 2 summarizes the outcomes arising in the game as function of β. Figures $1(a)$ and $1(b)$ show how π_R and π_M change as functions of β and when RPM and RRP prevail. The dashed blu line in Figure $1(a)$ represents π_M^{RPM}, i.e. retailer's profit if she has only some power in the determination of s but cannot refuse RPM. As expected, π_R^{RPM} is an increasing function of β. The red line in Figure $1(a)$ is π_R^{RRP}, which does not depend on β. The retailer's equilibrium profit, π_R^*, is the green line, with a kink and a discontinuity due to the change in the retail price regime. In Figure $1(b)$ we have drawn the manufacturer's profits: π_M^{RPM} (the blu line) is decreasing in β, π_M^{RRP} is constant (the red line), while the equilibrium profit π_M^* jumps upwards when the regime swaps from RRP to RPM. When buyer power is moderate, i.e. $\beta_1 \leq \beta \leq \beta_3$, we obtain an interesting outcome: RPM emerges endogenously, because both parties in the vertical channel have an incentive to eliminate the double markup. All solutions are however second best. The first best IV will not be reached. The manufacturer, due to the presence of buyer power, finds costly to convince the retailer to accept RPM and eliminate the double markup.

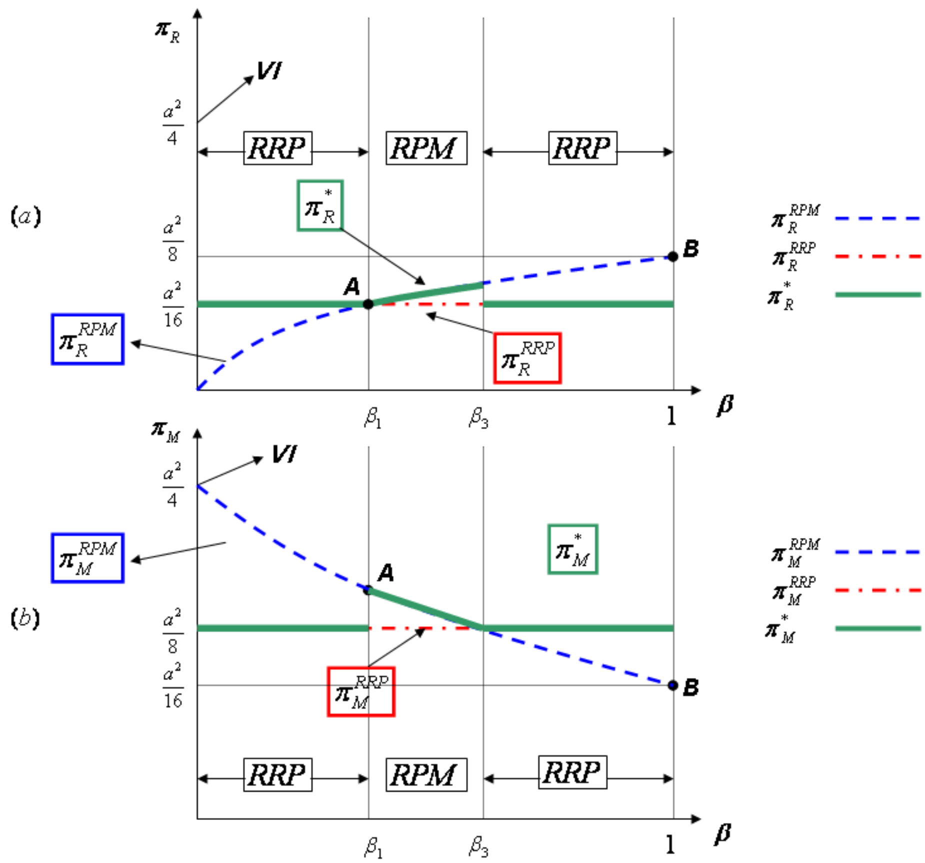

Figure 1: Retailer (a) and manufacturer (b) profits as function of β

6.3 Welfare analysis

In this section we study the welfare properties of the equilibrium vertical price restrictions. Figure 2 shows welfare under the two vertical price restrictions as a function of β. Equilibrium welfare, W^*, is given by the green line. It starts at a low level, very far from the first best ($W^{VI} = \frac{3}{8}a^2$), when buyer power is small and so RRP prevails. Welfare jumps upwards at β_1, because the change from RRP to RPM produces a social benefit. When buyer power is moderate, an endogenous RPM emerges, with positive social benefits, even if welfare does not achieve the first best. When the endogenous RPM prevails, welfare shrinks as buyer power increases, showing a negative relation between these two economic variables. A high buyer power leads to a swap in the retail price regime, from RPM to RRP, and, consequently, a

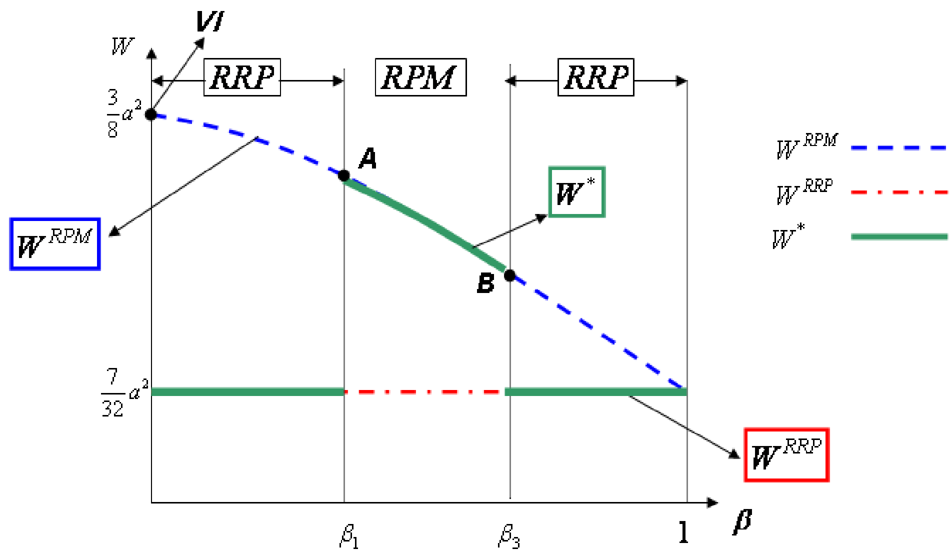

Figure 2: Welfare, buyer power and vertical price restrictions

low level of welfare. We have therefore proved the following result:

Proposition 5 *Welfare depends crucially on buyer power. When β is small (i.e. $0 \leq \beta < \beta_1$), RRP prevails and welfare is at its lowest level. When $\beta_1 \leq \beta \leq \beta_3$, RPM prevails and welfare is higher than under RRP but decreasing in β. When β is high, RPM and RRP give the same welfare, which is back to its lowest level.*

Proposition 5 points out that there is a non–monotonic relation between welfare and buyer power. When the latter is positive but small, welfare is low; when buyer power increases sufficiently, RPM is implemented endogenously and, consequently, welfare jumps up. But as β increases further, welfare decreases. When β is high, welfare returns to its minimum. Two factors contribute to explaining this non–monotonic relation: (1) double marginalization (when RRP prevails), (2) the predetermined unit discount given to the retailer, which affects the manufacturer's marginal cost. If buyer power is very small, both these distortions arise in equilibrium: the retailer increases the retail price above the manufacturer's recommended price and also gets a positive, though small, discount. If buyer power is sufficiently high, the first distortion disappears, since the retailer gives up retail price control, but she enjoys a higher unit discount. In this case welfare rises, given the elimination of double marginalization. However, if buyer power continues to grow, the unit discount keeps rising, making this second distortion very large. Welfare is thus maximal when buyer power is at an intermediate level (point A in Figure 2) when the first distortion is eliminated and the second one is reasonably low.[23]

6.4 Alternative orders of moves

For the sake of completeness we have studied alternative games characterized by different orders of moves. For instance, we have analyzed the inversion between the two intermediate stages. In this new game the recommended price is chosen by the manufacturer before knowing whether the retailer accepts his retail pricing contract proposal. Furthermore, in another game we have postponed the determination of s to the second stage, after deciding the retail pricing regime.[24]

By studying these two alternative games we realize that it is crucial, for an endogenous RPM to occur, that the retail pricing regime is set before $\overline{p}$. If this does not hold, RRP prevails. It is so because the retailer, being $\overline{p}$ already

[23] The case where s is so high that RPM leads to a lower welfare than RRP, as shown in Proposition 3, is never an equilibrium outcome in this model specification.

[24] We assume, instead, that the unit discount s is always determined before $\overline{p}$, because in reality, while s is determined in a long–run contract, $\overline{p}$ can be changed by the manufacturer at no cost more frequently.

determined, would accept RPM only when $\overline{p}$ is sufficiently low (to enjoy a sales increase). The manufacturer, however, is not willing to offer it under RPM because it would shrink his profit margin too much. When instead, as it is in our model, $\overline{p}$ is chosen after the retailer's decision concerning the retail pricing regime, the manufacturer can respond to a high s by rising $\overline{p}$.

7 Conclusions

The effect of buyer power on the equilibrium vertical price restriction has been investigated in a model based on the main features of two recent important antitrust disputes. It has been demonstrated that a retailer only accepts RPM in exchange for a high unit discount. For example, when the retailer's buyer power is very low, the retailer, receiving only a small unit discount, keeps control over the retail price (RRP prevails) and raises it above the recommended level (overpricing). This implies that double marginalization is not eliminated. When buyer power is very high, the manufacturer, knowing that the unit discount demanded by the retailer would be too high, will never propose a Maximum RPM. Only when the parties' bargaining power is balanced, the manufacturer offers an RPM contract and the retailer accepts it: under these circumstances an endogenous Maximum RPM arises. This represents the best situation for society because double marginalization is eliminated while the unit discount is not particularly high. Hence, in this context, the well known Galbraith's countervailing power hypothesis works.

Our conclusions question the current attitude of antitrust authorities on vertical price restrictions, which looks only at manufacturer's selling power according to a monotonically decreasing relationship: the lower the buyer power (i.e. the higher the manufacturer's seller power), the worse is the welfare effect of a vertical price restriction.[25] But we have shown that the best situation for welfare is not when the manufacturer has no selling power. We have also shown that equilibrium recommended prices reduce welfare in comparison with an endogenous RPM. This result casts some doubts about the current benevolent attitude of antitrust authorities towards recommended prices.

We have looked at a successive monopoly, where double marginalization and buyer power are relevant. It would be interesting, in future research, to extend the analysis to more complex vertical structures, where competition arises both upstream and downstream.

[25] For instance the EC block exemptions for vertical agreements state: "The market position of the supplier is the main factor in assessing possible anti–competitive effects of recommended or maximum resale prices. The stronger the supplier's position, the higher the risk that a recommended resale price or a maximum resale price is followed by most or all distributors". EC [2002], p. 26.

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APPENDIX

Proof of Lemma 1: Totally differentiating (2), we obtain:

$$y'(p)dp + y'(p)[ds + dp - d\overline{p}] + (p - \overline{p} + s)y''(p)dp = 0 \tag{A.1}$$

From the second order condition we know that: $G = 2y' + (p - \overline{p} + s)y'' < 0$. Using G in (A.1), we have: $Gdp = y'(p)(d\overline{p} - ds)$, and so

$$dp = \frac{y'(p)}{G}d\overline{p} - \frac{y'(p)}{G}ds \tag{A.2}$$

which gives the following implicit function:

$$p = \phi(\overline{p}, s) \tag{A.3}$$

where, from (A.2), we have:

$$\frac{\partial\phi(\overline{p}, s)}{\partial\overline{p}} = \frac{y'}{G} > 0 \quad \text{and} \quad \frac{\partial\phi(\overline{p}, s)}{\partial s} = -\frac{y'}{G} < 0$$

It is then obvious that:

$$\frac{\partial\phi(\overline{p}, s)}{\partial\overline{p}} = -\frac{\partial\phi(\overline{p}, s)}{\partial s} \tag{A.4}$$

The impact of a unit variation of $\overline{p}$ on p is exactly equal to that of a unit variation of s on p, with the opposite sign. Equation (A.4), from Euler's Theorem, also implies that: $\frac{\partial^2\phi}{\partial\overline{p}\partial s} = -\frac{\partial^2\phi}{\partial s^2}$ and $\frac{\partial^2\phi}{\partial s\partial\overline{p}} = -\frac{\partial^2\phi}{\partial\overline{p}^2}$. Last, to show that $\frac{\partial^2\phi}{\partial\overline{p}\partial s} = \frac{\partial^2\phi}{\partial s\partial\overline{p}}$, consider that

$$G = 2y'\left[\phi(\overline{p},s)\right] + \left[\phi(\overline{p},s) - \overline{p} + s\right] y''\left[\phi(\overline{p},s)\right] \tag{A.5}$$

with

$$\frac{\partial G}{\partial \overline{p}} = 2y''\frac{\partial \phi}{\partial \overline{p}} + \left(\frac{\partial \phi}{\partial \overline{p}} - 1\right) y'' + \left(p - \overline{p} + s\right) y'''\frac{\partial \phi}{\partial \overline{p}} \tag{A.6}$$

$$\frac{\partial G}{\partial s} = 2y''\frac{\partial \phi}{\partial s} + \left(\frac{\partial \phi}{\partial s} + 1\right) y'' + \left(p - \overline{p} + s\right) y'''\frac{\partial \phi}{\partial s} \tag{A.7}$$

Since $\frac{\partial \phi}{\partial s} = -\frac{\partial \phi}{\partial \overline{p}}$, we can write the latter as

$$\frac{\partial G}{\partial s} = -\left[2y''\frac{\partial \phi}{\partial \overline{p}} + \left(\frac{\partial \phi}{\partial \overline{p}} - 1\right) y'' + \left(p - \overline{p} + s\right) y'''\frac{\partial \phi}{\partial \overline{p}}\right] \tag{A.8}$$

Hence $\frac{\partial G}{\partial s} = -\frac{\partial G}{\partial \overline{p}}$. We can now analyze the cross partial derivatives of $\phi(\overline{p},s)$. We know that

$$\frac{\partial \phi}{\partial \overline{p}} = \frac{y'[\phi(\overline{p},s)]}{G(\overline{p},s)} \tag{A.9}$$

Hence:

$$\frac{\partial^2 \phi}{\partial \overline{p}\partial s} = \frac{y''\frac{\partial \phi}{\partial s}G - y'\frac{\partial G}{\partial s}}{G^2} \tag{A.10}$$

We know that

$$\frac{\partial \phi}{\partial s} = -\frac{y'[\phi(\overline{p},s)]}{G(\overline{p},s)}. \tag{A.11}$$

We can then compute

$$\frac{\partial^2 \phi}{\partial s\partial \overline{p}} = \frac{-y''\frac{\partial \phi}{\partial \overline{p}}G + y'\frac{\partial G}{\partial \overline{p}}}{G^2} \tag{A.12}$$

It follows that:

$$\frac{\partial^2\phi}{\partial\overline{p}\partial s} = \frac{-y''\frac{\partial\phi}{\partial\overline{p}}G + y'\frac{\partial G}{\partial\overline{p}}}{G^2} = \frac{\partial^2\phi}{\partial s\partial\overline{p}} \tag{A.13}$$

□

Proof of Lemma 3: The total differential of (7) is:

$$y'\left[\frac{\partial\phi}{\partial\overline{p}}d\overline{p} + \frac{\partial\phi}{\partial s}ds\right] + y'\frac{\partial\phi}{\partial\overline{p}}(d\overline{p} - ds) + (\overline{p} - s)\frac{\partial\phi}{\partial\overline{p}}y''\left[\frac{\partial\phi}{\partial\overline{p}}d\overline{p} + \frac{\partial\phi}{\partial s}ds\right] + \tag{A.14}$$
$$+(\overline{p} - s)y'\left[\frac{\partial^2\phi}{\partial\overline{p}^2}d\overline{p} + \frac{\partial^2\phi}{\partial\overline{p}\partial s}ds\right] = 0$$

From the second order condition we know that:

$$F = 2y'\frac{\partial\phi}{\partial\overline{p}} + (\overline{p} - s)\left[y''\left(\frac{\partial\phi}{\partial\overline{p}}\right)^2 + y'\frac{\partial^2\phi}{\partial\overline{p}^2}\right] < 0 \tag{A.15}$$

Thus, using F in (A.15), we get:

$$Fd\overline{p} + \left[y'\frac{\partial\phi}{\partial s} - y'\frac{\partial\phi}{\partial\overline{p}} + (\overline{p} - s)\frac{\partial\phi}{\partial\overline{p}}y''\frac{\partial\phi}{\partial s} + (\overline{p} - s)y'\frac{\partial^2\phi}{\partial\overline{p}\partial s}\right]ds = 0$$

But since $\frac{\partial\phi}{\partial s} = -\frac{\partial\phi}{\partial\overline{p}}$ and $\frac{\partial^2\phi}{\partial\overline{p}\partial s} = \frac{\partial^2\phi}{\partial s\partial\overline{p}} = -\frac{\partial^2\phi}{\partial\overline{p}^2}$ (from Lemma 1), then

$$Fd\overline{p} + \left\{-2y'\frac{\partial\phi}{\partial\overline{p}} - (\overline{p} - s)\left[y''\left(\frac{\partial\phi}{\partial\overline{p}}\right)^2 + y'\frac{\partial^2\phi}{\partial\overline{p}^2}\right]\right\}ds = 0 \tag{A.16}$$

Since the term in curly brackets in (A.16) is equal to $-F$ (as it emerges from (A.15)), it follows that (8) is true. Equation (8) implies that $\overline{p}^{RRP} = \kappa + s$, where κ is a strictly positive constant. Substituting (8) in (A.2) we have $dp^{RRP} = 0$. □

Proof of Proposition 1: Regarding the retailer's profit, it is easy to see that, with p^{RRP} independent of s, and since $\overline{p}^{RRP} - s = \kappa$, then from (1) it follows that π_R^{RRP} is also independent of s. For the manufacturer's profit, from (8), by the same argument, it is shown that π_M^{RRP} is independent of s. Hence, under RRP, the channel profit, defined as $\pi_M^{RRP} + \pi_R^{RRP}$, does not depend on s. Since both consumer surplus (Lemma 3) and channel profit are not affected by s, it follows that social welfare W (defined *à la Marshall* as the sum of consumer surplus and channel profit) is also independent of s. □

Proof of Lemma 4: Totally differentiating (9), we get:

$$y' d\overline{p} + (\overline{p} - s)y'' d\overline{p} + y'(d\overline{p} - ds) = 0 \tag{A.17}$$

Since $J = 2y' + (\overline{p} - s)y'' < 0$ to meet the second order condition, we can get from (A.17):

$$\frac{d\overline{p}^{RPM}}{ds} = \frac{y'}{J} > 0 \tag{A.18}$$

which leads to the following implicit function: $\overline{p}^{RPM} = \varphi(s)$, with

$$\varphi' = \frac{d\overline{p}^{RPM}}{ds} = \frac{y'}{J} = \frac{1}{2 + (\overline{p} - s)\frac{y''}{y'}} > 0 \quad \text{but} < 1.$$

Under RPM, as well as under RRP, an increase in s induces an increase in $\overline{p}$, but of a different magnitude. With a linear demand this is exactly half of that under RRP. □

Proof of Proposition 2: We only need to show the effect of s when the price constraint is binding. Since $p^{RPM} = \overline{p}^{RPM}$ and $\varphi' > 0$ (see Lemma 4), an increase in s leads to an increase in $\overline{p}^{RPM}$. Hence consumer surplus shrinks. Looking at the effect of s on the retailer's profit, we have $\pi_R^{RPM} = sy[\varphi(s)]$, and so, by differentiating it with respect to s, we get:

$$\frac{d\pi_R^{RPM}}{ds} = y + sy'\varphi' \tag{A.19}$$

Its sign depends upon s, being $y > 0, y' < 0$ and $\varphi' > 0$. The manufacturer's profit is: $\pi_M^{RPM} = [\varphi(s) - s]y[\varphi(s)]$. By differentiating it with respect to s we get:

$$\frac{d\pi_M^{RPM}}{ds} = (\varphi' - 1)\, y + (\overline{p}^{RPM} - s)y'\varphi' = (\overline{p} - s)y' < 0. \tag{A.20}$$

The last step in (A.20) is obtained by solving (9) for y and substituting it. Not surprisingly, the lower s is, the higher π_M^{RPM} is. Channel profit is now considered: $\pi_M^{RPM} + \pi_R^{RPM} = \varphi(s)y[\varphi(s)]$; differentiating it with respect to s, we get:

$$\frac{d(\pi_M^{RPM} + \pi_R^{RPM})}{ds} = \varphi'(y + y'\varphi) = \varphi' sy' < 0$$

The last step is again obtained by solving (9) for y and substituting it. Thus an increase in s reduces the channel profit: the loss to the manufacturer due to an increase in s is always higher than any gain obtained by the retailer. Since consumer surplus is also decreasing in s, we can state that welfare decreases with s. □

Proof of Proposition 3: First we evaluate $\frac{d\pi_R^{RRP}}{dp}$ when $p = \overline{p}^{RPM}$. From (2) we get:

$$\left.\frac{d\pi_R^{RRP}}{dp}\right|_{p=\overline{p}^{RPM}} = y + sy' = (2s - \overline{p}^{RPM})y' \quad \text{(A.21)}$$

The last step of the above expression is obtained by solving (9) for y and substituting it. The sign of $\frac{d\pi_R^{RRP}}{dp}\mid_{p=\overline{p}^{RPM}}$, since $y' < 0$, depends on the sign of $(2s - \overline{p}^{RPM})$. If $s < \frac{\overline{p}^{RPM}}{2}$ then $p^{RRP} > p^{RPM} = \overline{p}^{RPM}$. If $s = \frac{\overline{p}^{RPM}}{2}$ the two prices coincide (i.e. $p^{RRP} = p^{RPM} = \overline{p}^{RPM}$). If $s > \frac{\overline{p}^{RPM}}{2}$ then $p^{RRP} < p^{RPM} = \overline{p}^{RPM}$. Hence only if s is sufficiently small will the price under RRP be higher than under RPM. Consequently, consumer surplus under RRP is lower than under RPM. A high s changes the sign of this comparison.

Second, we analyze channel profit. It is easy to compute that

$$\frac{d(\pi_M^{RRP} + \pi_R^{RRP})}{dp} = y + py'$$

Computing this derivative when $p = p^{RPM} = \overline{p}^{RPM}$ we find

$$\left.\frac{d(\pi_M^{RRP} + \pi_R^{RRP})}{dp}\right|_{p=\overline{p}^{RPM}} = y + \overline{p}^{RPM}y' = sy' < 0 \quad \text{(A.22)}$$

Note that the last step in (A.22) is obtained by solving (9) for y and substituting it. It is also possible to compute $\frac{d(\pi_M^{RRP}+\pi_R^{RRP})}{dp}$ when $p = p^{RRP}$ (i.e. the price charged by the retailer under the RRP regime). In this case we get

$$\left.\frac{d(\pi_M^{RRP} + \pi_R^{RRP})}{dp}\right|_{p=p^{RRP}} = y + p^{RRP}y' = (\overline{p}^{RRP} - s)y' < 0 \quad \text{(A.23)}$$

This derivative is negative since $y' < 0$ and $\overline{p}^{RRP} > s$. Note that the last step is obtained by solving (2) for y and substituting it. Since the first order derivative of the channel profit with respect to p is negative both when

$p = p^{RPM}$ and when $p = p^{RRP}$, both prices are too high in comparison with the price that maximizes channel profit, provided that the latter is a strictly concave function. It follows that, under the two pricing regimes, decreasing p induces an increase in channel profit. Hence a decrease in p will lead to an increase in both consumer surplus and channel profit, and, consequently, in welfare. Hence, in order to compare the two pricing regimes from a welfare point of view, it is necessary to identify the regime charging the lower price. But the latter depends upon the sign of (A.21), and so on the level of s. $\square$