RJVs and Welfare with Knowledge Spillovers: A Dynamic Non-Tournament Model

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Abstract

This paper presents one of the first dynamic frameworks studying the relationship among Research Joint Ventures (RJVs), welfare, and knowledge spillovers. In modeling the dynamics we take three factors into account: the speed of technical progress, the size of the innovation, and the lenght of the disclosure lag necessary to enjoy the spillover. In the non-cooperative R&D regime, we identify three possible equilibria. They are classified, according to the timing of R&D investments, as early, intermediate, and late. The intermediate equilibrium is relevant because–besides providing new insights–it makes interesting the comparison between the non-cooperative game and the RJV regime. We show that RJVs dominate in welfare terms even in presence of low spillovers (differently from the literature following d'Aspremont and Jacquemin (1988) and Kamien *et al.* (1992)). Moreover we find that the type of industry in which the innovation is introduced and the size of the R&D output have an important impact in assessing RJV's welfare effect.

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1 Introduction

The growing importance of Research Joint Ventures (RJVs) in real world markets has produced a blooming theoretical literature concerning the individual and social incentives toward R&D cooperation.[1] These contributions have highlighted that knowledge spillovers are one of the most important determinants of RJVs.[2] However, spillovers exert two conflicting effects on the social desirability of RJVs. On the one hand, RJVs internalize the externality due to spillovers, but, on the other hand, they eliminate the incentive to invest in innovation to gain a competitive advantage in the product market. As pointed out by d'Aspremont and Jacquemin (1988) (henceforth DJ), the latter effect dominates the former if spillovers are low, and, consequently, RJVs become socially unappealing.[3] These findings have induced a general consensus that antitrust authorities, to grant a block exemption, should evaluate RJVs with discretion.[4]

These insights have been achieved using non-tournament models, which usually adopt a static approach (even if multi-stage), where firms can choose the *size* of their investments in innovation, while the *timing* remains out of the analysis.

We design a dynamic oligopoly model in which firms choose *when* to invest in a cost-reducing innovation. The R&D investment cost shrinks over time thanks to technical progress. We first analyze the equilibria under the non-cooperative regime and then we study the effects of a symmetric RJV agreement. When R&D is non-cooperative, we identify three possible equilibria, which are classified according to the timing of investment in innovation as: early, intermediate, and late.

In the early equilibrium, the competitive pressure induces firms to in-

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[1] See Caloghirou *et al.* (2003) for a survey on RJVs contribution (both theoretical and empirical). Some data extracted from the unofficial available databases (CATI, NCRA-RJV, CORE, and STEP-TWO RJVs) show that during the past couple of decades the number of RJVs has substantially grown, especially in high-technology areas (80% of actual RJVs) such as information technology, biotechnology and new materials.

[2] Recently, Hernan *et al.* (2003) provide an empirical support for the importance of spillovers in determining RJVs formation.

[3] In contrast, when the R&D stage is non-cooperative, a high spillover substantially reduces the individual competitive edge obtained through the introduction of an innovation.

[4] See Grossman and Shapiro (1986) and Motta (2004, ch. 4) for a discussion of the antitrust issues involved in a RJV.

vest soon in R&D, with the aim of exploiting the related cost advantage; in contrast, the late equilibrium is (implicitly) collusive: each firm delays the R&D investment, knowing that its innovative activity triggers its competitor's investment. (In this equilibrium, the R&D investment takes place only once the exogenous technical progress has sufficiently reduced its cost.) The intermediate equilibrium is an innovative feature of this paper, and is characterized by a competitive pressure weaker than the early one. This equilibrium is particularly relevant because it is the prevailing one for a large range of the parameters set, and because it makes interesting the comparison with the RJV regime.

We show that RJVs may improve welfare even in presence of low levels of spillovers (differently from DJ), that antitrust exemption is more (less) likely if the industry-specific technical progress is slow (fast), and that in the case of a major innovation there is less need to promote an RJV than in the case of a moderate innovation. All these relevant features cannot be dealt with by means of a static model.

We obtain the above results in a framework in which technical progress involves the reduction over time of innovation costs due to exogenous developments in basic research. Accordingly, projects that were unfeasible before, become viable at a given period.[5] Spillovers reduce the rival's innovation cost by a constant fraction. Hence, we are dealing with R&D inputs spillovers, as in Kamien *et al.* (1992) (henceforth KMZ).[6] In our set-up, the inter-firm spillover exerts its positive effect on the rival's innovation cost after a time period which we label "disclosure lag".

In our infinite horizon duopoly game, firms, in each period, are involved in a two–stage interaction: first they decide whether to innovate or not, and then they compete *à la* Cournot.

In the three equilibria arising in the market game, the role played by the competing firms are different. A leader emerges in the early and in

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[5] For instance, the research activity performed in a specific industry may be influenced by the technology adopted in other sectors. Geox, a well known brand in the footwear industry, is a good example of this effect: its innovations are due to the application of polyurethane developed in the aerospace industry.

[6] Amir (2000) compare DJ (1988), which is characterized by R&D output spillovers, with the R&D inputs spillovers in KMZ (1992). He suggests that−whenever the R&D sector is characterized by decreasing return to scale−the R&D input spillover assumption is sounder.

the intermediate equilibria, while in the late equilibrium firms invest at the same time. Two driving forces characterize the equilibria: the length of the follower's strategic delay, and the competitive pressure due to preemption, which leads to rent equalization (as in Fudenberg and Tirole (1985) and in Weeds (2002)).

As for the early equilibrium, the intuition can be summarized as follows. The innovation leader invests "very soon", because the second comer delays his decision to invest for a time longer than the disclosure lag. In fact−being the innovation cost still very high−he chooses to exploit both the cumulative effect of technical progress and the spillover. The length of the competitive advantage favors the innovation leader's payoff at the expenses of the second comer's one. Hence, to avoid preemption, the former invests soon. In contrast, a late equilibrium arises once technical progress has substantially reduced the innovation costs, so that an innovation leader cannot emerge, because the rival would immediately copy her decision: with low R&D cost, the spillover effect becomes necessarily negligible. In this case, any innovator – anticipating that there will be no leadership – waits until her choice maximizes the joint discounted stream of net profits.

However, in our model a third type of equilibrium arises: the intermediate one, where the innovator knows that the second comer will exploit the spillovers as soon as the relevant information is obtained, i.e. exactly at the end of the disclosure lag. Hence, the leader's competitive advantage period is shorter than in the early equilibrium. This harms the leader's discounted profits, while benefiting the follower. Therefore, in this case, the competitive pressure−that leads to rent equalization−is weaker and does not force the leader to invest "too soon". Consequently, the adoption dates are intermediate between the ones obtained in the very competitive early equilibrium and those obtained in the late, cooperative, one.

By applying the sub-game perfection criterion, we can select the unique equilibrium for the overall game. A moderate innovation size acts against the early equilibrium. This happens because a small cost reduction implies a weak incentive to innovate first: in this case, when the spillover is high, the unique sub-game perfect equilibrium is the intermediate one. In fact, the follower, grasping large benefits from investing at the end of the disclosure lag, select this strategy for a long time interval. The leader finds it convenient not to wait until the "immediate reply" strategy becomes optimal for the follower, which gives rise to the intermediate equilibrium. When the spillover is low, the unique equilibrium is the late one, because the "immediate reply" strategy for the follower becomes optimal earlier, which allows for higher leader's payoffs in this equilibrium.

In contrast, when the innovation size is large, an early equilibrium may emerge, because a major innovation, bringing a large cost advantage to the leader, enhances the incentive to be first. However, the higher the spillover, the earlier is the second comer optimal investment date in reply to an early leader's investment. This reduces the leaders' efficiency advantage period, leading to the dominance of the intermediate equilibrium. Notice also that an high spillover increases the second comer's payoff in the intermediate time interval, softening the preemption incentive to invest for the leader. This milder competition implies higher equilibrium payoffs for both firms.

In the RJV regime, the two firms form a unique R&D department, taking the spillovers into account: they jointly decide when to invest in R&D; afterwards, they compete in the Cournot stage. The result of a welfare-improving RJV in the presence of low spillovers is important, because the costs related to the innovation activity (i.e. what is usually referred to as R&D costs), also involve outlays for some training of the employees, the purchase of some new equipment, the expenses for marketing startup, etc. Hence, the knowledge spillover parameter is very likely to be low.[7]

The RJV solution is welfare improving either when it anticipates the R&D activity in comparison with that arising in the non-cooperative game or if it begins the R&D activity after the first innovator of the market game but sufficiently before the second comer.

RJVs are dominated by the early equilibrium, because the intensity of competition is very high in this non–cooperative game, so that the innovator is forced to anticipate a lot the investment decision. In contrast, RJVs always dominate the late equilibrium, where firms exhibit a collusive-like behavior and postpone the R&D investments. More interestingly, RJVs tend to dominate the intermediate equilibrium, because the internalization effect of RJVs is likely to dominate the mild competition effect that characterizes this equilibrium.

The speed of the technical progress influences the RJVs dominance over

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[7] For the same argument, it is unlikely that an RJV cartelization, as modeled in KMZ (1992), can significantly increase the inter-firm spillovers.

the non–cooperative regime. If the industry is in its technological maturity, R&D costs decrease slowly over time, so that firms tend to postpone the investments. If instead the industry is experiencing a technical revolution, R&D costs decrease rapidly, and firms tend to anticipate their investments. Thus, the fast decrease in R&D costs improves the leader's payoff, and makes competition more fierce. Accordingly, in this case, an RJV becomes welfare improving only for higher spillover levels: an increase in the spillover delays the intermediate equilibrium because it improves the second comer's result and hence softens competition.

Similarly, in the case of a major innovation, the threshold level of spillovers making RJV dominant is higher than in case of a moderate innovation. The competitive advantage granted by a major innovation is larger than that arising with a moderate one, and this leads the innovator to anticipate the investment date. Consequently, there is less (more) need to promote an RJV in the case of a major (moderate) innovation.

Related work Our paper is related to the non–tournament class of models on RJV, which flourished after DJ's seminal contribution. Suzumura (1992) generalized their results, Salant and Shaffer (1998) show the existence of optimal asymmetric strategies in the R&D stage. However, their approach−generating asymmetries in profits−calls for compensations, such as side payments, among the RJV members. Motta (1992) presents a model that enlarges DJ contribution to the product innovation case in a vertical differentiation context. In his model, the cut–off spillover level required to obtain welfare enhancing RJVs is higher than that identified by DJ. Poyago–Theotoky (1999) investigates a model in which firms can choose the level of the spillover, and shows that under the RJV solution the firms select a full disclosure strategy. KMZ (1992), besides using inputs spillovers in the R&D process, distinguish the R&D cartel from the RJV cartel. In the former, firms coordinate their R&D activities, while in the latter they also enjoy a significant increase in spillovers. KMZ contend that RJV cartelization always dominate the market solution in terms of welfare.

All of these contributions are based on the static approach. This paper, to the best of our knowledge, is the first attempt to investigate the welfare effects of RJVs in a dynamic oligopoly with spillovers.[8] Fudenberg and Ti-

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[8] Ordover and Willig (1985) present a dynamic model in which two firms decides when

role (1985) provide the methodology to analyze the equilibria in dynamic duopolies and prove their existence. Their approach has been developed by Hoppe and Lehmann–Grube (2005). Katz and Shapiro (1987) introduce the idea that the second comer may have lower R&D costs, even if, in their model, he imitates immediately the innovator decision. All these contributions adopt a dynamic approach but do not investigate RJVs.

The paper proceeds as follows. In Section 2 we present our framework, and in Section 3 we discuss the equilibrium concept adopted in the analysis and we compute the different market equilibria, in which firms compete both in the innovation and in the product stages. Then, sub-game perfectness is invoked as a selection device among market equilibria. In Section 4 we identify the equilibrium under the RJV regime, in which firms jointly select the period to innovate, and compete only in the market stage. Then, we rank the sub-game perfect non–cooperative equilibrium and the RJVs solution in welfare terms. Concluding comments in Section 5 end the paper. The proofs of the Propositions are provided in the Appendix.

2 The model

2.1 The production stage and its welfare implications

Following DJ (1988), KMZ (1992) and most of the related literature, we consider an industry composed of two firms, 1 and 2, selling an homogeneous good and competing *à la* Cournot. Market demand is linear and equal to: $P = a - bQ$, where P is the market clearing price and $Q = q_1 + q_2$ is the total quantity supplied. Each firm has a unit cost of production c.

The original part of our analysis concerns the innovation stage, and specifically the R&D cost, that changes over time. In each period t firm i ($i = 1, 2$) decides whether to invest in R&D or not. This investment immediately yields a cost-reducing process innovation, which shrinks the unit production cost by an amount x, with $x < c$.[9] Hence firm i's post–

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to enter a new product into the market, in the presence of a leader. They compare the non–cooperative solution with the RJV outcome and show that RJVs should be judged by antitrust authorities under the "rule of reason" approach. However they have no spillover effect.

[9] The functional forms and dynamics for the firms' R&D costs will be modeled in Section 2.2.

innovation production cost is $C(q_i) = (c - x)q_i$. Clearly, each firm's payoff will depend not only on its adoption date but also on its rival's one.

Firms compete with an infinite horizon, and discount future profits at the common rate r. If both firms have not invested up to period t, their individual profits in the Cournot sub-game at t coincide with those of the pre–innovation stage, i.e.

$$\pi_i^{00} = \frac{A^2}{9b}, \tag{1}$$

where $A = a - c$ and the superscript $\{00\}$ indicates that both firms do not innovate at t. The instantaneous welfare (computed *à la Marshall*) is then equal to:

$$W^{00} = \frac{4}{9}\frac{A^2}{b}. \tag{2}$$

Expression (2) is obtained by adopting a "second best" welfare criterion: as in Suzumura (1992), we compute the welfare obtained under Cournot. In fact, we assume that the oligopolistic competition in the second stage quantity game lies beyond the regulatory power of the government, i.e. firm cannot be forced to sell at marginal cost while they can play Cournot.[10]

If instead only one firm, say firm 1, invests in R&D at t, it benefits of an efficiency advantage, and obtains a higher market share. The market price at t decreases in comparison with the pre-innovation level, while the individual profits become:

$$\pi_1^{10} = \frac{(A + 2x)^2}{9b}; \pi_2^{10} = \frac{(A - x)^2}{9b}, \tag{3}$$

where $\{10\}$ indicates that firm 1 has invested in R&D while firm 2 has not. Notice that $\pi_1^{10} > \pi_2^{10}$, $\pi_1^{10} > \pi_1^{00}$ and $\pi_2^{10} < \pi_2^{00}$. Since$-$as it can be easily verified$-q_2^{20} = \frac{A-x}{3b}$, to preserve the duopolistic structure characterizing our market we need to introduce:

Assumption 1: $A > x$.

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[10] Had we accepted the more conventional criterion, according to which what matters is the welfare obtained in a first best situation, little would have changed. What is important for our results is that social welfare, however measured, calls for an early adoption of the new cost-reducing process.

This hypothesis implies that, in a Cournot environment, the cost-reducing innovation is non−drastic (see, Denicolò (1996)). In case of asymmetric behavior at t, welfare is:

$$W^{10} = \frac{8A(A+x) + 11x^2}{18b}, \tag{4}$$

with $W^{10} > W^{00}$.

Finally, we need to compute the outcomes when both firms have innovated at instant t. In this case they produce a higher quantity with respect to the *status quo* because they are more efficient; therefore, the market price is lower. Individual profits at t are:

$$\pi_i^{11} = \frac{(A+x)^2}{9b}, \tag{5}$$

where the superscript $\{11\}$ indicates that both firms have innovated.

Obviously, $\pi_1^{10} > \pi_1^{11}$; notice, moreover, that the difference between π_1^{10} and π_1^{11} is increasing in x: the innovator's profits are more than proportional to her cost advantage, which enlarges her market share and increases the price to cost margin.

When both firms have innovated, the social welfare is:

$$W^{11} = \frac{4(A+x)^2}{9b}. \tag{6}$$

with $W^{11} > W^{10}$ (by Assumption 1). In our model, it might be that firms simultaneously invest in R&D, so that individual profits rise from (1) to (5) and welfare jumps from (2) to (6). Alternatively, firms may behave asymmetrically, so that there are both an innovation leader and a follower. Under these circumstances individual profits first increase from to (1) to (3) (and welfare from (2) to (4)) and then from (3) to (5) (and welfare from (4) to (6)).

2.2 R&D costs

In our set-up, only one research project is available to the firms. For the first firm investing in R&D, the innovation cost evolves over time according to the following equation:

$$C_1(t_1) = \gamma x e^{-\rho(t_1 - t_0)}, \quad \text{for} \quad t_1 \in [t_0, \infty), \tag{7}$$

where t_1 is the calendar time when the first firm has introduced the innovation. Hence, we are assuming that the innovation becomes technically feasible at time t_0 at a cost, γx, which then decreases at the constant rate $\rho \geq 0$ thanks to advances in pure research and to the availability of new results obtained in related fields. Of course, this form of technical progress is exogenous to any single firm.[11] We assume that the initial cost is very high so that no firm finds profitable to innovate at t_0. It is clear, from (7), that, if a firm innovates in period t, R&D costs are sunk at that time.

As for the second firm introducing the innovation, the cost evolution is described by the following equation:

$$C_2(t_2) = \begin{cases} \gamma x e^{-\rho(t_2 - t_0)} & \text{for} \;\; t_2 \in [t_1, t_1 + \Delta) \\ (1-\theta)\gamma x e^{-\rho(t_2 - t_0)} & \text{for} \;\; t_2 \in [t_1 + \Delta, \infty) \end{cases} , \tag{8}$$

where $\theta \in [0, 1)$ is a spillover parameter and Δ is the delay that is needed to grasp the benefit stemming from the rival's innovative activity, i.e. Δ represents an exogenously determined disclosure lag.[12] Equation (8) describes a situation in which the innovation is only partially appropriable: whenever $\theta > 0$, the second comer enjoys a reduction in R&D costs by imitating his competitor at $t_2 \geq t_1 + \Delta$. Hence, for any $\theta \in [0, 1)$ the innovation follower enjoys an inter-firm spillover. Notice, however, that it takes time to imitate an innovation: in his classic study, Mansfield (1985) reports that in 61% of cases the innovator's rival knows the relevant information in less than eighteen months. An obvious but important consequence of our assumption is that the introduction of an innovation grants a cost advantage (and hence higher profits) for a time period equal to Δ. Clearly, our assumption stylizes an extremely simple form of spillover. It would have been preferable to consider a stochastic inter-firm spillover, in which the probability of information diffusion depends upon the time elapsed from the introduction

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[11] Alternatively, one might think that ρ stylizes a market dimension effect due to the increase in aggregate income: an increase in profits per period lowers the innovation cost in relative terms.

[12] Clearly, it would be preferable to endogenize the spillover parameter, and the length of the disclosure lag. In principle, this is possible: for example, as for θ, we could follow Jin and Troege (2006), which suggest that θ can be raised by firms, paying a convex imitation cost. Nevertheless, we preferred not to pursue these potential developments of the model, because our framework, due to its dynamic nature, is already fairly complex: any further complication would have required a much heavier use of numerical techniques.

of the innovation and on the follower's imitation effort. However, even the simplest stochastic formulation—namely the one involving a constant probability of information diffusion—precludes the attainment of explicit results. Moreover, we do not judge a constant probability of information disclosure an improvement upon our formulation, since the sparse empirical evidence available tells us that the probability of successful imitation increases over time. Hence, our formulation has been chosen as the optimal compromise between analytical tractability and "realism".

In what follows we will often restrict the values for Δ. In particular, we now introduce the following technical assumption:

Assumption 2 $\Delta \leq \bar{\Delta} = \frac{1}{r} \ln \left(1 + \frac{r}{\rho} \frac{2A+3x}{6A+3x}\right)$.

The purpose of Assumption 2 is to limit the number of cases that we need to consider. To verify that Assumption 2 does not restrict Δ to values too short to be sensible, we compute $\bar{\Delta}$ when x approaches 0 (since this choice lowers $\bar{\Delta}$), the annual interest rate is 0.03, and $\rho = \{0.01,\ 0.05,\ 0.09\}$.[13] With these values, $\bar{\Delta}$ becomes, respectively, equal to $\{23.105,\ 6.077,\ 3.512\}$. Hence, the restriction implied by Assumption 2 is realistic in most contexts. From the vantage point of economic analysis, a low Δ is interesting, because it makes the role of the inter-firms spillovers more important.

3 The market equilibria

In this Section we discuss the equilibria in the non-cooperative R&D game. To this purpose, we first explain the equilibrium concept adopted to solve the model. Because in our model, the payoff functions will not, in general, be single-peaked, we need to deal with the existence of multiple equilibria. We shall divide time in three sub-intervals, in such a way that in each interval the equilibrium is unique. We will then select the globally unique equilibrium referring to the concept of sub-game perfectness.

3.1 The equilibrium concept

As already mentioned, in our set-up only one research project is available to the firms: hence, the choice to innovate at time t_i is an irreversible stopping

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[13] These values for ρ have a relevant economic interpretation that will become apparent later.

decision. Therefore, our model belongs to the class of symmetric timing games, which can be divided into two sub-classes, depending upon which firm (the one that moves first or the one that moves second) obtains the higher payoff.

We can make this point more precise, by assuming for the moment that we have exogenously assigned the task of moving first to one of the two firms. In this case, there is a first mover advantage if the firm that must move first obtains the higher payoff. If, instead, the first mover obtains the lower payoff, there is a second mover advantage. Obviously the first mover is assumed to behave optimally, choosing the innovation time that maximizes its payoff, given the second mover optimal choice.

To deal with first mover advantage games, we drop the hypothesis of exogenously assigned roles and we follow Hoppe and Lehman-Gruber (2005) assuming that:

Assumption 3: Time is continuous in the sense of discrete but with a grid that is infinitely fine.

Assumption 4: If the two firms are indifferent between being the first or the second mover at any date t, then the role of the leader is played by the firm with a female CEO[14] and the role of the follower is played by the other firm, which is run by a male CEO.[15]

Assumption 3 circumvents the difficulties of applying backward induction in continuous time and allows us to deal with strategies with arguments that are continuous variables (the adoption times) "as if" their arguments were discrete variables. Assumption 4 is used to rule out the possibility of coordination failures as an equilibrium outcome. In other words, in our model firms do not choose to move at the same instant of time if they know that they would regret this choice afterwards. As Hoppe and Lehman-Gruber (2005) remark, an equilibrium involving coordination failures cannot be obtained in the case of a continuous-time game without a grid, in which equilibria are defined to be the limits of discrete-time mixed strategies (Fudenberg and Tirole, (1985) and (1991)). By contrast, in the limit of a discrete-time

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[14] Say firm 1, which will henceforth be referred to as if it were a female.

[15] Hoppe and Lehman-Gruber (2005) actually assume that - when the two firms wish to invest on the same date - there is a probability equal to 0.5 that either of them invests while the other becomes the follower. Clearly, the difference between the two alternative assumptions is of no consequence.

game in which the period length converges to zero, coordination failure is a possible equilibrium outcome. Since we use a discrete-time game with "extremely short" time lags to represent "continuous time" for expositional ease, we find it natural to introduce Assumption 4 to rule out the possibility of coordination failures.[16]

The logic to obtain the unique sub-game perfect equilibrium in first-mover advantage games can be described by exploiting Panel (a) in Figure 1. The payoff function $V_1(t_1, T_2^*)$ gives firm's 1 net profits when she invests at time t_1, while the rival invests at time T_2^*; these profits are discounted back to time t_0 for convenience. $V_2(t_1, T_2^*)$ gives firm's 2 discounted payoff when he invests at time T_2^*, while the first invests at time t_1. Because $V_1(t_1, T_2^*)$ is single-peaked at $t_1 = T_1^*$, the first firm would like to move first at T_1^*. But the roles of innovation leader and follower are not pre-assigned. Hence, when the second firm knows that the other will adopt at time T_1^*, it is in his interest to preempt at time $T_1^* - dt$. By backward induction, we conclude that the equilibrium strategy for the first innovator is to invest as soon as the leader's payoff is equal to the follower's one (i.e. at $\underline{T}_1$). (Assumption 4 grants us that the first innovator is actually firm 1.) Notice that the "preemption argument" spelled out above yields equal payoffs to the two firms in the sub-game perfect equilibrium. Hence, in this case the equilibrium involves what is often referred to in the literature as "rent dissipation".

[Figure 1 about here]

In dealing with "second mover advantage" games, we rely again on Hoppe and Lehman-Gruber's analysis. In this case, they assume that the equilibrium is driven by expectations and make the following hypothesis:

Assumption 5: Whenever the innovation leader payoff is lower than the second comer's, firm 1 believes that firm 2 never enters first.

The logic to obtain the unique sub-game perfect equilibrium in this case can be explained by means of Panel (d) in Figure 1. $V_1(t_1, T_2^*)$ is single-peaked at $t_1 = T_1^*$, moreover, the $V_1(t_1, T_2^*)$ curve lies below the $V_2(t_1, T_2^*)$

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[16] A complementary way to justify Assumption 4 is as follows: consider two firms that are ex-ante identical in every aspect except for an infinitesimal difference in the time needed to implement the innovation decision. In this more realistic world, the firm with the faster implementation period would emerge as the innovation leader because, when it starts finding convenient to sunk the R&D cost, the other is not ready yet to become the leader.

curve for any $t_1 \leq T_1^*$. Hence firm 1 chooses $t_1 = T_1^*$ (the date granting her the highest possible payoff) while no firm has an incentive to preempt its rival before date t_1.

Assumption 5 (and therefore the equilibrium it implies) may seem arbitrary. In fact, it rules out the mixed-strategies equilibria, often referred to as a war of attrition (Fudenberg and Tirole (1991)). However−if we reject Assumption 5−our firms start to randomize at T_1^*. Hence, Assumption 5 reduces the expected time to the first innovation, thus operating against our preferred result, which is the one of a welfare improving RJV.

Assumptions 4 and 5 guarantee that in each of our three sub-intervals, there is a unique equilibrium.

3.2 Alternative market equilibria

In the next Sub-sections, we divide the time line into three sub-intervals, in which three different equilibria may arise. First, a situation may emerge in which an innovation leader decides to invest very early, so that the follower's optimal strategy is to wait more than Δ periods before imitating the leader. The features of this early equilibrium will be analyzed in Sub-section 3.2.1.

We then consider the equilibrium that arises when the innovation leader delays her innovation so that the follower's optimal choice is to invest exactly Δ periods after the leader, in order to grasp the inter-firm spillover as soon as this is possible. We shall refer to this solution as the intermediate equilibrium, which will be analyzed in Sub-section 3.2.2.

Finally, the innovation leader may decide to invest "very late". In this case, the R&D cost is so low that it is optimal for the second firm to immediately enter upon the rival's investment, without exploiting the inter-firm spillover. An equilibrium with these characteristics is labeled the late one and it will be discussed in Sub-section 3.2.3.

We denote by $V_1(t_1, t_2)$ the discounted stream of future profits obtained by the first firm investing at t_1 while her rival sinks the innovation cost at t_2, that is:

$$V_1(t_1, t_2) = \int_{t_0}^{t_1} \pi_1^{00} e^{-r(t-t_0)} dt + \int_{t_1}^{t_2} \pi_1^{10} e^{-r(t-t_0)} dt + \\ + \int_{t_2}^{\infty} \pi_1^{11} e^{-r(t-t_0)} dt - C_1(t_1) e^{-r(t_1-t_0)}. \tag{9}$$

Accordingly, the second firm's payoff is:

$$V_2(t_1, t_2) = \int_{t_0}^{t_1} \pi_2^{00} e^{-r(t-t_0)} dt + \int_{t_1}^{t_2} \pi_2^{10} e^{-r(t-t_0)} dt + \int_{t_2}^{\infty} \pi_2^{11} e^{-r(t-t_0)} dt - C_2(t_2) e^{-r(t_2-t_0)}. \tag{10}$$

3.2.1 The early equilibrium

By investing early, the leader incurs a high innovation cost (equation (7)), because "pure research" has not yet provided many results upon which to build upon. The high innovation cost is the reason why the follower prefers to invest with a delay longer than Δ years: in fact, if he waits more than Δ, he not only nets the benefits from imitation, but he can also grasp relevant additional gains from pure research, which is still producing results that are quantitatively important for reducing the innovation cost. When the leader sinks the costs at $t_1 \leq T_2^* - \Delta$, the follower's payoff-maximizing choice is to invests at

$$T_2^* = t_0 - \frac{1}{\rho} \ln \left(\frac{4A}{9b\gamma(r+\rho)(1-\theta)} \right), \tag{11}$$

which is obtained by maximizing (10) with respect to t_2.

Hence, notice that the time interval in which an early equilibrium exists is $t_1 \in [t_0, T_2^* - \Delta]$.

The comparative statics on T_2^* gives sensible results. The higher the inter-firms spillover, the sooner the second comer invests: a high θ reduces—*ceteris paribus*—the follower's costs and therefore anticipates his investment date. An increase in A or a decrease in b induce an expansion in per-period profit and hence they anticipate the second comer's decision to innovate; an increase in γ and r delays his investment decision, because the innovation is more costly or the future profits are more heavily discounted. The technical progress parameter ρ plays a twofold role: on the one side, at any date t_2, the innovation costs are lower, which calls for an earlier investment; on the other side, a faster reduction in innovation costs may induce a firm to wait because it knows that the cost will quickly become smaller.[17]

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[17] With a low spillover, the first direct effect prevails over the second indirect one; in

Given the follower's investment timing, the leader—when computing the payoffs yielded by an early investment—must balance the profits stemming from the cost advantage (which lasts for a time span longer than Δ) with her higher innovation cost.

When θ is low, T_2^*, and hence $T_2^* - \Delta$, are large because, as already remarked, a low θ, implying an high follower's costs, postpones his optimal investment date. Hence, with a low θ, the leader's payoff function displays in the interval $t_1 \in [t_0, T_2^* - \Delta]$, the typical inverted U shape obtained by Fudenberg and Tirole (1985) and by Weeds (2002) (panel (a) in Figure 1). In fact, an increase in the leader's adoption time induces a reduction in her innovation cost which is large when t_1 is close to t_0; hence the cost reduction more than compensates for the shortening in her efficiency advantage period. However, as time goes by, the beneficial effect on the leader's payoff induced by the technological externality becomes quantitatively less important, and the negative effect due to the reduction in her efficiency advantage period becomes dominant, inducing a decrease in the first innovator payoff. The inverted U shape for the payoff function calls for a unique leader's adoption time.

In the Appendix, we show that this date, which is obtained by maximizing the innovation leader's payoff function (9) with respect to t_1, is:

$$T_1^* = t_0 - \frac{1}{\rho} \ln \left(\frac{4(A+x)}{9b\gamma(r+\rho)} \right) \tag{12}$$

From (11) and (12), it is clear the inequality $T_1^* \leq T_2^* - \Delta$ is satisfied only for $\theta \leq \theta'(\Delta) \equiv 1 - \frac{A}{A+x} e^{\rho\Delta}$, i.e. for a "low" θ.

Moreover, the weak inter-firm spillover tend to make the first innovator payoff larger than the second mover's one (i.e. $V_1(T_1^*, T_2^*) \geq V_2(T_1^*, T_2^*)$). As discussed in the Appendix, a few calculations show that this happens when $\theta \leq \tilde{\theta} \equiv 1 - \frac{A}{A+x} \left[1 + \frac{4xr}{\rho 3(2A+x) + r(2A-x)} \right]^{\frac{\rho}{r}}$.

Hence, when $\theta \leq \min\{\tilde{\theta}, \theta'(\Delta)\}$ (i.e. in area A in Figure 2), the qualitative behavior of the payoff functions (9) and (10) is the one depicted in panel (a) in Figure 1.

[Figure 2 about here]

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contrast, when θ is high, the impact of an increase in ρ on T_2^* may well be positive for realistic parameter values.

In this case the sub-game perfect equilibrium is obtained applying the preemption argument: by backward induction, we conclude that the equilibrium strategy for the first innovator is to invest as soon as the leader's payoff is equal to the follower's one ($\underline{T}_1 \leq T_1^*$ in Figure 1, panel (a)).

When the inter-firm spillover parameter θ is above the threshold $\theta'(\Delta)$ the first innovator's payoff does not reach a maximum in the interval $[t_0, T_2^* - \Delta]$. In fact, an high inter-firm spillover, reducing the follower's costs, anticipates his timing for the investment (equation (11)). However, the inter-firms spillover may be still low enough that the first innovator's payoff, when she invest at $T_2^* - \Delta$, is higher than the second comer discounted net profits computed at his optimal reply date, T_2^*. This happens when $\theta \leq \theta''(\Delta) \equiv 1 - \frac{4Are^{(r+\rho)\Delta}}{(r+\rho)(6A+3x)(e^{r\Delta}-1)+4Ar}$. Hence, when $\theta'(\Delta) < \theta \leq \theta''(\Delta)$, i.e. in area B in Figure 2, there is a first mover advantage in some right interval of $T_2^* - \Delta$ (Figure 1, panel (b)), and the preemption argument applies. Again, the equilibrium involves the first adoption at $\underline{T}_1$.

In area C in Figure 2, (i.e. for $\theta > \max\{\theta'(\Delta), \theta''(\Delta)\}$) the first innovator's payoff is always increasing in the interval $[t_0, T_2^* - \Delta]$ (because $T_1^* > T_2^* - \Delta$); moreover, the spillover parameter θ is high enough that, at $T_2^* - \Delta$, the first innovator's payoff is lower than the second comer's one computed at T_2^* (Figure 1, panel (c)). The increasing payoffs imply that it is in each firm's interest to wait until $T_2^* - \Delta$; moreover, the preemption argument sketched above cannot apply. Hence, there is no equilibrium before $T_2^* - \Delta$.

Finally, when θ is below $\theta'(\Delta)$ but above $\tilde{\theta}$, (area D in Figure 2) there is a maximum for the first innovator's payoff in the interval $[t_0, T_2^* - \Delta]$, but at that maximum (T_1^* in Figure 2, panel (d)) the leader's discounted stream of profits is lower than the follower's one. Hence, firm 1 waits up to T_1^*; at this time she invests (by Assumption 5) granting a second mover advantage to her rival, who invests at $T_2^* - \Delta$.

The possible existence of a second mover advantage equilibrium associates our model also to the one by Katz and Shapiro (1987).

The above arguments are formally presented in:

Proposition 1

When Assumption 2 is satisfied, for $t_1 \in [t_0, T_2^* - \Delta]$

(a) if $\theta \leq \min\{\tilde{\theta},\ \theta'(\Delta)\}$ the unique sub-game perfect equilibrium is $\{\max\{\underline{T}_{1,}t_0\}, T_2^*\}$, where $\underline{T}_1$ is the earliest adoption date for the first firm such that, $V_1(\underline{T}_1, T_2^*) \geq V_2(\underline{T}_1, T_2^*)$.

(b) if $\theta'(\Delta) < \theta \leq \theta''(\Delta)$, the unique sub-game perfect equilibrium, is $\{\max\{\underline{T}_{1,}t_0\}, T_2^*\}$.

(c) if $\theta > \max\{\theta'(\Delta),\ \theta''(\Delta)\}$, no equilibrium exists in the interval $[t_0, T_2^* - \Delta)$.

(d) if $\tilde{\theta} < \theta \leq \theta'(\Delta)$, the unique sub-game perfect equilibrium is $\{T_1^*, T_2^*\}$.

Proof: See the Appendix.

3.2.2 The intermediate equilibrium

We now consider what happens when the innovation leader invests after $T_2^* - \Delta$.

When $t_1 > T_2^* - \Delta$, the follower's choice is among to copy immediately, to wait less than Δ, and to wait Δ before investing (to grasp the inter-firm spillover).[18] We define as $\bar{T}$ the first date when the payoff obtained by the second firm by "immediately following" the leader innovation, becomes as high as the payoff granted to the follower by the decision of waiting Δ periods before investing in R&D.

It turns out that, when $t_1 < \bar{T}$, the follower's optimal choice is to wait exactly Δ periods before innovating. The intuition for this behavior is clear: when $t_1 < \bar{T}$ the innovation cost is still high enough that it is convenient for the follower to let the innovation leader enjoy an efficiency advantage for Δ periods in exchange for a reduction in his own R&D costs, caused by the spillover.

Again, the nature of the equilibrium depends on the values taken by Δ and by the spillover parameter θ.

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[18] Waiting more than Δ can never be optimal for the follower, just because such strategy calls for an investment at T_2^* as a reply to a leader's investment set up at $t_1 \in [t_0, T_2^* - \Delta]$, which is not the case we are studying.

A high θ (i.e. $\theta \in (\theta'''(\Delta), 1)$[19]), grants to the follower a second mover advantage for a large sub-interval of $t_1 \in (T_2^* - \Delta, \bar{T}]$. In fact, if t_1 is not too larger than $T_2^* - \Delta$, the leader bears an high innovation cost. Hence, the large spillover induces a second mover advantage because its relevant magnitude, more than compensates the first innovator's efficiency advantage. Notice that the presence of a high spillover makes the leader's payoff, at $T_2^* - \Delta$, lower than the follower's one. When t_1 is high (i.e. not too far from $\bar{T}$), the innovation cost is low, due to the technological externality: this weakens the role of the inter-firms spillover, which leads to a first mover advantage. This case is portrayed in Figure 3, Panel (a), which clearly suggests that the unique equilibrium is $\{\hat{T}_1, \hat{T}_1 + \Delta\}$, with $\hat{T}_1$ being the maximum for $V_1(t_1, t_1 + \Delta)$ in the interval $t_1 \in (T_2^* - \Delta, \bar{T}]$.

[Figure 3 about here]

When $\theta \in (\theta''(\Delta), \theta'''(\Delta)]$, the spillover parameter θ is sufficiently low to guarantee the existence of a first mover advantage in a large portion of the interval $(T_2^* - \Delta, \bar{T}]$. This case is depicted in Figure 3, panel (b): the reduction in θ shifts the payoff function for the follower downward, inducing the existence of a preemption equilibrium at the intersection point T_1^{ip}, where the payoff for the two firms are identical. In fact, in this equilibrium the advantage in production costs, enjoyed by the leader for Δ periods, is exactly compensated by the lower R&D costs granted to the innovation follower by the joint effects of the inter-firm spillover and of the technological externality. Due to the technological externality, the leader's payoff is increasing in her adoption time at least up to the equilibrium date, hence, she never wants to invest earlier than T_1^{ip}, while she is forced not to invest later than T_1^{ip} by the usual preemption argument. From the follower's perspective, a Δ periods delay is optimal, because the R&D cost has not yet reached a level low enough to call for a faster than Δ response to the leader's innovation.

When $\theta \in (0, \theta''(\Delta)]$, the imitation benefit is small and the first firm payoff is always larger than that of second firm's one. This allows us to apply the usual preemption argument, which moves back the equilibrium to the early stage described in the previous Sub-section. Figure 3, panel (c) depicts the behavior for the payoffs functions in this case.

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[19] The threshold $\theta'''(\Delta)$ will be defined later. Bear in mind that a bit of algebra shows that $\theta'''(\Delta) > \theta''(\Delta)$, as depicted in Figure 2.

The above arguments are summarized in:

Proposition 2

Let $\bar{T} = t_0 - \frac{1}{\rho}\ln\left(\frac{4A(1-e^{-r\Delta})}{9br\gamma[1-(1-\theta)e^{-(r+\rho)\Delta}]}\right)$, and
$\theta'''(\Delta) = \min\left\{1, 1 - \frac{[4Ar+\rho(6A+3x)](e^{-r\Delta}-1)+r(2A+x)}{re^{-(r+\rho)\Delta}[4(A+x)-(2A+3x)e^{-r\Delta}]}\right\}$.
When Assumption 2 is satisfied, in the interval $t_1 \in (T_2^* - \Delta, \bar{T}]$,

(a) if $\theta \in (\theta'''(\Delta), 1)$, the unique sub-game perfect equilibrium is $\{\hat{T}_1, \hat{T}_1 + \Delta\}$, where:

$$\hat{T}_1 = t_0 - \frac{1}{\rho}\ln\left(\frac{4(A+x)-(2A+3x)e^{-r\Delta}}{9b\gamma(r+\rho)}\right);$$

(b) if $\theta \in [\theta''(\Delta), \theta'''(\Delta)]$ the unique sub-game perfect equilibrium is $\{T_1^{ip}, T_1^{ip} + \Delta\}$, where:

$$T_1^{ip} = t_0 - \frac{1}{\rho}\ln\left(\frac{3(2A+x)(1-e^{-r\Delta})}{9br\gamma[1-(1-\theta)e^{-(r+\rho)\Delta}]}\right). \tag{13}$$

(c) if $\theta \in [0, \theta''(\Delta))$ no equilibrium exists in the interval $(T_2^* - \Delta, \bar{T}]$.

Proof: See the Appendix.

When we focus on $\bar{T}$, we realize that the comparative statics is qualitatively similar to the one characterizing T_2^*, except for the effect of the inter-firm spillover. In fact, an increase in θ increases $\bar{T}$: a more relevant benefit from imitation postpones the undertaking of a line of action that prescribes the forsaking of the benefit itself. More interestingly, we see from (13) that in case (b) an increase in the inter-firm spillover delays the equilibrium. In fact, an higher θ benefits the follower and hence weakens the competition. This happens because the equilibrium $\{T_1^{ip}, T_1^{ip} + \Delta\}$ is preemptive: the first innovator sinks the R&D costs as soon as her payoffs becomes larger than the rival's: a higher payoff for the second comer therefore softens the incentive to invest for the leader and hence softens the competition.

We conclude this Sub-section by jointly discussing the results obtained in Propositions 1 and 2, which allow us to select the equilibrium in the entire region $[t_0, \bar{T}]$ for most parameters configurations.

	$x = 0.05$		
	$\rho = 0.01$	$\rho = 0.05$	$\rho = 0.09$
$\theta'(0)$	0.048	0.048	0.048
$\theta''(\Delta)$	0.174	0.107	0.095

	$x = 0.50$		
	$\rho = 0.01$	$\rho = 0.05$	$\rho = 0.09$
$\theta'(0)$	0.333	0.333	0.333
$\theta''(\Delta)$	0.286	0.195	0.177

Table 1: Cut–off values for a unique intermediate equilibrium

When $\theta \geq \max\{\theta'(\Delta), \theta''(\Delta)\}$, which is, in area C of Figure 2, the intermediate equilibrium exists (Proposition 2, parts (a) and (b)) and no equilibria exists in the interval $[t_0, T_2^* - \Delta]$ (Proposition 1, part (c)). Since $V_1(t_1, T_2^*)$ is increasing in the whole interval $t_1 \in [t_0, T_2^* - \Delta]$ (Figure 1, panel (c)), we can safely conclude that the intermediate equilibrium is the unique one in the interval $[t_0, \bar{T}]$.

When $\theta \leq \theta''(\Delta)$ (i.e. in the lower portion of area A and in area B of Figure 2), no intermediate equilibrium exists in the interval $(T_2^* - \Delta, \bar{T}]$ (Proposition 2, part (c)), while Proposition 1, parts (a) and (b) grants the existence of a unique pure strategy preemption equilibrium in $[t_0, T_2^* - \Delta]$, which is the unique equilibrium in $[t_0, \bar{T}]$.

In the remainder of area A and in area D, (i.e. for $\theta'(\Delta) < \theta < \theta''(\Delta)$) both an early and an intermediate equilibrium exist (Proposition 1, parts (a) and (d) and Proposition 2, parts (a) and (b)). In this case, we need to select the unique global equilibrium exploiting the sub-game perfectness criterion (an analysis that will be carried out later).

To appreciate the quantitative importance of the restrictions needed for the intermediate equilibrium to be unique, notice that$-$for $\Delta \in [0, \bar{\Delta}]$ $-$ $\theta''(\Delta)$ is increasing while $\theta'(\Delta)$ is decreasing. Hence, when Assumption 2 is satisfied, and θ is above the higher of the two values reported in each column of Table 1, the intermediate equilibrium is the unique one.

3.2.3 The late equilibrium

Finally, if the innovation leader decides to invest "late" (i.e. when $t_1 \in [\bar{T}, \infty)$) the R&D cost is so low that it is optimal for the second firm to immediately enter upon rival's investment, without exploiting the inter-firm spillover.

In this case the first firm is aware that—as soon as she innovates—the second firm will "immediately" follow her decision and invest. Hence, each firm takes her decision anticipating such a follower's behavior. This leads to an equilibrium where the two firms maximize their joint payoff: knowing that it will be immediately followed, each firm delays its innovation until its discounted sum of profits reaches its maximum. In this context, where firms remain symmetric, the maximization of a single firm's payoff coincides with the joint maximization.

Formally, the innovation leader's behavior is summarized by the following proposition.

Proposition 3

When Assumption 2 is satisfied, for $t_1 \in [\bar{T}, \infty)$,

(a) if $\theta \in [\hat{\theta}(\Delta), 1]$ where $\hat{\theta}(\Delta) = 1 - e^{(r+\rho)\Delta}\left[1 - (r+\rho)\frac{1-e^{-r\Delta}}{r}\frac{4A}{2A+x}\right]$, both firms invest at $\bar{T}$;

(b) if $\theta \in [1 - \frac{r+\rho}{r}e^{\rho\Delta} + \frac{\rho}{r}e^{(r+\rho)\Delta}, \hat{\theta}(\Delta))$, both firms invest at:

$$T^{le} = t_0 - \frac{1}{\rho}\ln\left(\frac{2A+x}{9b\gamma(r+\rho)}\right). \tag{14}$$

(c) if $\theta \in [0, 1 - \frac{r+\rho}{r}e^{\rho\Delta} + \frac{\rho}{r}e^{(r+\rho)\Delta})$, the sub-game perfect equilibrium is either T^{le} or it does not exists.

Proof: See the Appendix.

When the spillover is low, the scope for waiting Δ before investing is limited and hence $\bar{T}$ is low. Therefore, for a low θ the payoff-maximizing choice for the adoption time is unconstrained and thus the late equilibrium is given by (14).

3.3 Equilibrium selection

The analysis developed in the previous Sub-sections allows to conclude that in our model three equilibria may arise. Sub-game perfectness requires that the equilibrium must survive all of the possible off-equilibrium deviations. Accordingly, in the present context the equilibrium selection must be based on the comparison of the innovation leader's payoff at any adoption date earlier than the one that is part of the candidate equilibrium. Unfortunately, this task cannot be performed analytically, due to the high degree of non linearity in our model. Hence, we now present some numerical results.[20]

In our simulations, we normalize to unity the market dimension parameter A, and we fix the discount rate r to 0.03, which is consistent with computing calendar time in years. The parameter γ does not play any substantial role: the effect of an higher γ (i.e. of a less efficient R&D) is to postpone all of the equilibria, without changing their relative convenience. Likewise, the choice for b is inconsequential: an increase in b always induces a proportional contraction in per period profits. Hence, we choose $b = 1$ and $\gamma = 20$ with no loss of generality. As for ρ, we study—in the schumpeterian tradition—industry-specific rates of reduction in innovation costs. The technologically mature industry I is still benefiting from some technical progress in the sectors producing its machinery. Accordingly, $\rho = 0.01$. In industry II, $\rho = 0.05$, which is the case of a fairly dynamic sector. Finally, industry III is a sector involved in a "technical revolution", and $\rho = 0.09$. To appreciate our figures, bear in mind that the average economy-wide increase in productivity is of the order of 2% a year; moreover consider that, in a specific sector, faster-than-average technical progress may well go together with an above-average increase in wages. In this case the cost-effective technical progress parameter, ρ, is lower than the productivity growth rate.

To preserve the duopolistic structure of our market, we consider only non-drastic innovation (Assumption 1). Hence, the size of the R&D output, x, is lower than A ($x < 1$). We investigate two types of innovative output: a moderate innovation where $x = 0.05A (= 0.05)$ and a major innovation where $x = 0.5A (= 0.5)$.

Figure 4 portrays the equilibria arising in the case of a moderate innovation. Panel (a) highlights that in Industry I a low spillover gives rise, for a given Δ, to a late equilibrium, while as the spillover increases the intermediate equilibrium prevails. For instance, when $\Delta = 2$, (refer to Table 2) the late equilibrium prevails when $\theta \leq 0.047$, while if $\theta > 0.047$ we have the

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[20] Our routine has been written in Gauss, and it is based on a discretization of the space $[\theta \text{ x } \Delta]$, for $\theta \in [10^{(-10)}, 0.9]$ and $\Delta \in [10^{(-10)}, 3]$. We have used 240.000 gridpoints, however our results do not relevantly change for any number of evaluation points larger than 15.000. This routine is available upon request from the authors.

intermediate equilibrium.

[Figure 4 about here]

The intuition for this result is the following: as underscored by Fudenberg and Tirole (1985), the smaller the cost reduction, the weaker is the incentive to innovate first.[21] Hence, a small x means that the highest deviation payoff for an early innovator is low. When $x = 0.05$, the early equilibrium never prevails over the late one in a direct comparison. Moreover, a low spillover gives rise to a late equilibrium because it shrinks the intermediate region, since the second firm has a weak incentive to wait the time necessary to enjoy a modest R&D cost-reducing spillover (refer to the definition for $\bar{T}$ and to Figure 3, panel (c)). Hence, the late equilibrium prevails over any possible deviation occurring in the intermediate period.

Industry	**Innovation**			
	moderate		**major**	
I	$\theta \leq 0.047$	late	$\theta \leq 0.101$	early
	$\theta > 0.047$	intermediate	$\theta > 0.101$	intermediate
II	$\theta \leq 0.056$	late	$\theta \leq 0.104$	early
	$0.056 < \theta \leq 0.063$	early		
	$\theta > 0.063$	intermediate	$\theta > 0.104$	intermediate
III	$\theta \leq 0.056$	late	$\theta \leq 0.131$	early
	$0.056 < \theta \leq 0.079$	early		
	$\theta > 0.079$	intermediate	$\theta > 0.131$	intermediate

Table 2: R&D equilibria ($\Delta = 2$)

When θ grows, the intermediate region enlarges, leading to a situation in which the first firm's deviation payoff becomes larger than her late equilibrium payoff. This leads to the prevalence of the intermediate equilibrium.

Panel (b) in Figure 4 shows the equilibria arising in Industry II. Again, for a given Δ, if the spillover is very low, the equilibrium in the R&D stage is the late one, for the same reasons as explained before. However, as θ increases (but it is still lower than $\theta''(\Delta)$), the early equilibrium prevails. This happens in the small areas contained between the two curves exiting from the origin in Figure 4 (refer also to Table 2). To understand this result,

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[21] This happens because the single innovator profit function, π_1^{10}, is more convex in x than π_1^{11} (see Sub-section 2.1)

bear in mind that an increase in ρ raises the payoffs in the intermediate region, because the R&D costs are lower.[22] The increase in the deviation payoff in the intermediate region destroys the late equilibrium, and moves the equilibrium to the early stage, because $\theta-$being lower than $\theta''(\Delta)-$is still small enough so that the intermediate equilibrium does not exists (refer to Sub-section 3.2.2).

Finally, a further increases in θ (above $\theta''(\Delta)$), drives us into the region in which the intermediate equilibrium exists; moreover, an increase in θ, reducing the first innovator's payoff in the early stage, makes the intermediate equilibrium dominant.

Figure 4, panel (c) shows the equilibrium selection in Industry III ($\rho = 0.09$): we have the same pattern observed for Industry II, with the only difference being that the θ threshold that discriminates the intermediate equilibrium from the early one is higher. This happens because the payoffs in the early region are higher than in the intermediate one because the former ones benefit more from rapid technical progress.

The case of a major innovation is portrayed in Figure 5. There, $x = 0.50A(= 0.50)$. Here, the late equilibrium never prevails: a high x favors the selection of the early equilibrium. However, in our framework, an early equilibrium arises only for moderate values of the spillover parameter. In fact, when θ increases so that the intermediate equilibrium exists, the latter prevails for two reasons. First, a high θ negatively influences the first innovator payoffs in the early interval, because it anticipates the follower's investment date (equation (11)). Second, in the intermediate interval, as the spillover increases, the second comer's payoff gets larger, softening the incentive to invest for the leader. This milder competition implies higher payoffs for both firms, inducing the selection of the intermediate equilibrium.

[Figure 5 about here]

In sum, our analysis of the equilibrium selection process suggests that the intermediate equilibrium is the sub-game perfect one in large portions of the parameter space.

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[22] The effect of ρ on the late equilibrium payoff is of course similar, but it is less significant since at that time the R&D cost are already very low.

4 Research joint venture

Having characterized the market equilibria, we are now ready to analyze the effects of an RJV. In our model, an RJV is set up by the firms with the aim of maximizing the sum of their discounted profits, taking into account the spillovers; the instruments for such a joint profit maximization are the innovation dates in the R&D stage.

In dealing with an RJV, we introduce some specific hypotheses. First, the two firms must introduce the innovation at the same time: no asymmetry in the R&D equilibrium is allowed, because differences in the times of adoption imply an asymmetry in profits and hence call for side payments.[23]

Second, the information advantage obtained by firms engaging in an RJV is the same that is obtained by the second entrant when she waits Δ. In terms of KMZ (1992), we do not distinguish between R&D cartelization and RJV cartelization. Clearly, this is a debatable assumption: an RJV should grant not only a faster but also an easier−and hence less costly−information flow when compared with a decentralized solution. However, if we interpret the innovation costs in an extensive way, incorporating into them the costs for the training of the employees required by the new production process, for some new machineries (or for adaptation of the existing plant), and so on, we are led to think that the spillover parameter, besides being relatively low, cannot be significantly increased by an RJV cartelization.

In this framework, the objective function for the RJV is:

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[23] The hypothesis concerning the symmetry in RJV's has been questioned by several papers in the literature, and most notably by Salant and Shaffer (1998). Building on the fact that a mean-preserving spread in the marginal costs raises industry profits in Cournot competition, they show that firms may have an incentive to deviate from a symmetric RJV agreement; moreover Salant and Shaffer argue that this may be the case in the classic DJ (1988) example. However more recently, Leahy and Neary (2005) have compared the conditions required for an asymmetric profit-maximizing cooperative outcome with those needed for the stability of the non-cooperative equilibrium. They have found that−in the DJ case−once the equilibrium stability conditions are considered, the asymmetric RJV outcome prevails for a fairly small range of parameters. In our framework, an asymmetric RJV outcome is profit maximizing only for relatively low values of the spillover parameter (i.e. when $\theta < 8x/(2A + 5x)$). Moreover, when this parameter restriction is satisfied, the symmetric RJV optimal timing is between the two optimal adoption dates obtained for the two firms in an asymmetric RJV agreement. Hence, our result would not change significantly if we allowed for an asymmetric RJV.

$$V^{RJV}(T^{RJV}) = 2\left[\int_{t_0}^{T^{RJV}} \pi_i^{00} e^{-r(t-t_0)} dt + \right. \tag{15}$$

$$\left. + \int_{T^{RJV}}^{\infty} \pi_i^{11} e^{-r(t-t_0)} dt \right] - (2-\theta)\gamma x e^{-(r+\rho)(T^{RJV}-t_0)}.$$

The problem is to maximize (15) with respect to T^{RJV}.

The RJV solution is:

$$T^{RJV} = t_0 - \frac{1}{\rho} \ln\left(\frac{(2A+x)}{9b\gamma(r+\rho)} \frac{2}{2-\theta}\right) \tag{16}$$

Notice that an increase in the spillover parameter θ anticipates the symmetric RJV optimal timing, simply because firms perceive that their (joint) R&D cost is lower.

4.1 Welfare-improving RJV

It is now time to show under which parameter restriction an RJV is welfare improving. The high degree of non-linearity characterizing our model asks for a numerical analysis. Therefore, to evaluate when a symmetric RJV agreement can be welfare improving, we compute numerically−for the same parameter configurations used in the previous Section−the social welfare attained by an RJV and by the sub-game perfect market equilibrium.

We find that an RJV is always welfare enhancing in the portion of the parameter space where a late equilibrium prevails. This is hardly surprising. In a late equilibrium firms maximize their payoffs, disregarding the consumer's surplus, which is instead incorporated into the social welfare function. Hence, in this equilibrium, from the social planner's perspective, firms invest too late. Comparing (16) with (14) it is easy to notice that, for $\theta \in [0,1)$ the symmetric RJV solution anticipates the late market equilibrium, and therefore it is welfare-improving.

In contrast, the RJV solution always prescribes to invest later than the follower's innovation date in an early equilibrium (compare (16) with (11)), and hence its adoption would damage the social welfare when it is the early equilibrium that prevails.

In the case of the intermediate equilibrium our calculations shows that an RJV tends to be welfare-improving—when compared to the market solution—for large portions of the parameter space. In fact, the RJV optimal investment timing is often between the innovation leader's entry date and the second comer's entry date. In this case, the symmetric RJV agreement tends to prevail because it implies an immediate jump of the instantaneous welfare from W^{00} to W^{11}(RJV efficiency effect), while the market solution implies that the welfare stops for Δ periods at W^{10}. Since $W^{11} > W^{10}$, because production is more efficient, the faster innovation diffusion implied by an RJV bears valuable welfare consequences.

Figure 6 portrays the case of a moderate innovation ($x = 0.05$). In every panel (i.e. for each type of industry) the area contained between the Δ axis and the lower line exiting from the origin represents the parameter configurations where an RJV is welfare-enhancing. This happens because a low spillover parameter implies that the late equilibrium prevails.

[Figure 6 about here]

As θ increases, the type of industry matters, because in industry I, an early equilibrium never arises (see Figure 4, panel (a)), while for industry II ($\rho = 0.05$) and for industry III ($\rho = 0.09$), this type of equilibrium prevails in a (small) parameter area (see Figure 4, panel (b) and (c)). Accordingly, in these areas, the RJV agreement is welfare damaging. However, the regions where an RJV harms the social welfare are much larger than the regions where—in Figure 4—the early equilibrium was sub-game perfect. For some (moderate) values of θ, the RJV adoption timing is larger than the follower's innovation date in the intermediate equilibrium. Here, we have a welfare-lessening RJV coupled with an intermediate equilibrium. As the spillover gets stronger, the intermediate equilibrium timings increases, (refer to eq. (13)), while the RJV adoption date is reduced (eq. (13)). When the RJV timing is between the leader's and the follower's innovation date in the intermediate equilibrium, our symmetric RJV readily becomes welfare-improving due to the RJV efficiency effect.[24]

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[24] This is why an RJV can be welfare damaging also when $\rho = 0.01$: in this case, even if the early equilibrium does not exist, an RJV prescribes a too late adoption when compared with the intermediate equilibrium.

Table 3 shows the thresholds levels of θ where society is better off with the RJV regime and with $\Delta = 2$. It is worth emphasizing that the sparse existing evidence (Mansfield *et al.* (1981), Mansfield (1985), Hernan *et al.* (2003)) leads us to think that – on average – the disclosure lag is relatively short (less than 18 months). Hence, the choice for the disclosure lag value used to obtain the thresholds in Table 3, acts against our preferred result, which is the one of a welfare improving RJV.

From Table 3, we notice that, as ρ gets higher, we require a larger inter-firm spillover for an RJV to be welfare improving. More rapid exogenous technical progress makes *ceteris paribus* the market solution more appealing. An increase in ρ anticipates the intermediate equilibrium more than the RJV one. In fact, a higher ρ reduces the innovation cost, thus increasing the leader's payoffs more than the follower's, making competition more fierce. Therefore, for an RJV to be welfare improving, higher values for the spillover parameter are required: these increase the follower's payoff, therefore softening competition and compensating for the effect of technical progress.

[Figure 7 about here]

Figure 7 considers the case of a major innovation ($x = 0.50$): an RJV cannot be welfare improving for low spillover levels, because the late equilibrium is never sub-game perfect (refer to Figure 5). The threshold values are shown in Table 3, again for $\Delta = 2$.

Industry	**Innovation**	
	moderate	**major**
I	$\theta \leq 0.047$ $\theta \geq 0.146$	$\theta \geq 0.149$
II	$\theta \leq 0.056$ $\theta \geq 0.270$	$\theta \geq 0.284$
III	$\theta \leq 0.056$ $\theta \geq 0.376$	$\theta \geq 0.403$

Table 3: Welfare improving R&D ($\Delta = 2$)

Our analysis points out an important result concerning the spillover level that induces a welfare improving RJV. While in the existing literature it is commonly accepted that cooperation in RJV makes society better off only when inter-firms spillovers are high (i.e. $\theta > 0.5$), we have shown that in a dynamic framework the RJV regime also dominates the non-cooperative solution when spillovers are low. Moreover, we argue that factors such as the type of industry in which the innovation is introduced and the size of the R&D output have an important impact on the welfare effect of the RJV. With regard to the type of industry, the area in which the RJV dominates gets larger as we move toward sectors in the technological maturity. In contrast, society needs fewer RJVs when industries experience a technical revolution. If instead the size of the R&D output is considered, the area in which an RJV dominates is smaller in the case of a major innovation (in comparison with a modest one).

In sum, our policy implications are the following. First, we have confirmed that−as in static models−RJV exemptions must be granted by Antitrust Authorities with discretion. This happens not only because it is necessary to consider industry–specific spillover levels (as in DJ), but, above all, because there does not exist a unique threshold level for RJV dominance in every industry. Second, we have shown that antitrust exemption depends negatively upon the size of the innovation, and that it is positively affected by the speed of technical progress.

5 Conclusions

In this paper, in contrast to the previous static literature, we have analyzed the social incentives to form an RJV in a dynamic framework. This approach seems appropriate given the many dynamic features of the innovative activity. We investigate a model in which R&D costs are decreasing over time and are influenced by inter-firms knowledge spillovers, and in which firms are involved in each period in a two stage interaction. First they decide whether to invest in R&D or not, and then they compete *a là* Cournot.

We find that, when R&D is non-cooperative three equilibria with different timings of investment may arise, and we have labeled them as early, intermediate and late. In the early and in the intermediate equilibria we have innovation diffusion, while in the late one firms invest at the same time. The two variables that characterize the equilibria are the competitive pressure due to preemption and the length of the follower's delay.

We find that for a large interval of spillover levels, the intermediate equilibrium is sub-game perfect, making particularly interesting the comparison with the RJV regime. Concerning this issue, our main results are that RJVs may improve welfare even in presence of low spillovers, that the beneficial effects of RJVs are more (less) likely if the industry is in its technological maturity (start-up), and when the innovation is moderate. The antitrust policy implications of these results are the dividend paid by our dynamic approach.

Our results have been obtained using a deterministic inter–firm spillover. Ideally, we would like to introduce a stochastic spillover lag, influenced not only by the time elapsed from the first innovator's decision, but also by the endogenous second comer's imitation effort. However, this setting makes the model intractable. Moreover, we have analyzed only one dimension of the R&D investment decision: the timing. Our investigation has considered as exogenous the other dimension, which is the size of the investment. However, just by introducing the temporal dimension, we have been forced to perform some numerical computation to select the equilibrium and to assess the dominance between the two regimes. Hence, our formulation has been chosen as the optimal compromise between analytical tractability and "realism".

Finally, an RJV should grant not only a faster (as in our model) but also an easier information flow in comparison with the non–cooperative regime: this implies that our results are valid in a framework rather unfavorable for a research coalition.

This contribution is limited to the case of process innovation: the analysis of the environment in which a product innovation can be introduced in the market is left for future research.

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APPENDIX

Proof of Proposition 1

Proof of part (a). As with Fudenberg and Tirole (1985) and Weeds (2002) we start characterizing the optimal strategy for the follower. When the first firm has sunk the innovation cost at time $t_1 \in [t_0, T_2^* - \Delta]$, the payoff at time t_0 for the second firm, when it invests at t_2, is given by (10).

The second comer may decide to wait at least Δ to grasp the inter-firms spillover; in which case the innovation cost relevant in (10) is $C_2(T_2) = (1-\theta)\gamma x e^{-\rho(T_2 - t_0)}$, and a few straightforward calculations show that T_2^* maximizes $V_2(t_1, t_2)$.

Alternatively, the second comer could decide not to wait for Δ periods, and in this case he should invest at:

$$T_2' = t_0 - \frac{1}{\rho} \ln \left(\frac{4A}{9b\gamma(r+\rho)} \right). \tag{A.1}$$

This second alternative obviously requires that $T_2' \in [t_1, t_1 + \Delta)$. Had the latter restriction not been satisfied, the innovation follower would have benefited from the spillover. Since $T_2' > T_2^*$, whenever $t_1 \in [t_0, T_2^* - \Delta]$ the innovation follower grasps the imitation benefits and invests at T_2^*. Because the innovation follower optimal decision is T_2^*, his payoff can be written as:

$$\begin{aligned} V_2(t_1, T_2^*) &= \frac{A^2}{9br} + \left[-\frac{(2A-x)x}{9br} \right] e^{-r(t_1 - t_0)} \\ &+ \frac{\rho}{r(r+\rho)} \frac{4Ax}{9b} \left(\frac{4A}{9b\gamma(r+\rho)(1-\theta)} \right)^{\frac{r}{\rho}}, \end{aligned} \tag{A.2}$$

which implies: $\frac{\partial V_2(t_1, T_2^*)}{\partial t_1} > 0$, and $\frac{\partial^2 V_2(t_1, T_2^*)}{(\partial t_1)^2} < 0$ in the whole interval $[t_0, T_2^* - \Delta]$. Also note that $\frac{\partial V_2(t_1, T_2^*)}{\partial \theta} > 0$ for every $t_1 \in [t_0, T_2^* - \Delta]$.

Having determined the optimal decision for the follower, we now determine the leader's best strategy. When $t_1 \in [t_0, T_2^* - \Delta]$, the innovation leader payoff is given by (9) in which the innovation costs are provided by (7) and $t_2 = T_2^*$.

Exploiting equation (11), we obtain:

$$\begin{aligned} V_1(t_1, T_2^*) &= \frac{A^2}{9br} + \left[\frac{4(A+x)x}{9br} - \gamma x e^{-\rho(t_1-t_0)}\right] e^{-r(t_1-t_0)} \\ &\quad - \frac{(2A+3x)x}{9br}\left(\frac{4A}{9b\gamma(r+\rho)(1-\theta)}\right)^{\frac{r}{\rho}}. \end{aligned} \tag{A.3}$$

Hence, $\frac{\partial V_1(t_1,T_2^*)}{\partial t_1} \gtreqless 0$ when $t_1 \lesseqgtr T_1^*$ (with T_1^* given by 12). Notice that T_1^*, in general, need not be smaller than $T_2^* - \Delta$. Notice also that $\frac{\partial V_1(t_1,T_2^*)}{\partial \theta} < 0$ for every $t_1 \in [t_0, T_2^* - \Delta]$.

When $\theta \leq \theta'(\Delta)$, it is easy to show that $T_1^* \leq T_2^* - \Delta$, just because the latter inequality requires $A + x \geq \frac{A}{1-\theta} e^{\rho\Delta}$, and hence $\theta \leq 1 - \frac{A}{A+x} e^{\rho\Delta}$. We now check whether – when $t_1 = T_1^*$– the leader's payoff is larger than the follower's one. Exploiting equation (A.3), $V_1(T_1^*, T_2^*)$ can be easily written as:

$$V_1(T_1^*, T_2^*) =$$
$$= \frac{A^2}{9br} + \frac{4\rho(A+x)x}{9br(r+\rho)}\left(\frac{4(A+x)}{9b\gamma(r+\rho)}\right)^{\frac{r}{\rho}} - \frac{(2A+3x)x}{9br}\left(\frac{4A}{9b\gamma(r+\rho)(1-\theta)}\right)^{\frac{r}{\rho}},$$

while, from (A.2), $V_2(T_1^*, T_2^*)$ is:

$$V_2(T_1^*, T_2^*) =$$
$$= \frac{A^2}{9br} - \frac{(2A-x)x}{9br}\left(\frac{4(A+x)}{9b\gamma(r+\rho)}\right)^{\frac{r}{\rho}} + \frac{\rho}{r(r+\rho)}\frac{4Ax}{9b}\left(\frac{4A}{9b\gamma(r+\rho)(1-\theta)}\right)^{\frac{r}{\rho}}.$$

A few calculations allow us to show that $V_1(T_1^*, T_2^*) \geq V_2(T_1^*, T_2^*)$ if $\theta \leq \tilde{\theta}$. Notice moreover that $\tilde{\theta} \leq \theta'(\Delta)$ for $\Delta \in [0, \Delta']$, where $\Delta' = \frac{1}{r}\ln\left[1 + \frac{r4x}{\rho 3(2A+x) + r(2A-x)}\right] > 0$. Hence, $V_1(T_1^*, T_2^*) \geq V_2(T_1^*, T_2^*)$ and $T_1^* < T_2^* - \Delta$ if $\theta \leq \min\{\tilde{\theta},\, \theta'(\Delta)\}$.

To conclude that the first firm equilibrium adoption date is $\max\{\underline{T}_1, t_0\}$, we follow the argument developed in Fudenberg and Tirole (1985): when $V_1(T_1^*, T_2^*) > V_2(T_1^*, T_2^*)$, it is in each firm's interest to adopt at time T_1^* if the other firm has not adopted up to that time. But if a firm knows that the other will adopt at time T_1^*, it is in its interest to preempt at time $T_1^* - dt$, whenever $V_1(T_1^* - dt, T_2^*) \geq V_2(T_1^*, T_2^*)$. By backward induction, we conclude that the equilibrium strategy for the first innovator is $\max\{\underline{T}_1, t_0\}$, where $\underline{T}_1$ is the earliest adoption date such that $V_1(\underline{T}_1, T_2^*) \geq V_2(\underline{T}_1, T_2^*)$. In this case no firm wants to invest and anticipate the other to avoid be preempted later on. To check that the region $\theta \leq \min\{\tilde{\theta},\, \theta'(\Delta)\}$ is non-empty, we need to show that $\tilde{\theta} > 0$. Notice that $\tilde{\theta} > 0$ requires that $\left[1 + \frac{4xr}{\rho(6A+3x)+r(2A-x)}\right]^{\frac{\rho}{r}} < \frac{A+x}{A}$; notice moreover that $\left[1 + \frac{4xr}{\rho(6A+3x)+r(2A-x)}\right]^{\frac{\rho}{r}} < \left[1 + \frac{r}{\rho}\frac{4x}{6A+3x}\right]^{\frac{\rho}{r}}$. The right hand side of the last inequality takes values, for $\rho/r \in [0, \infty)$, that belong to the interval $[1, e^{4x/(6A+3x)}]$. Because the maximum in this interval, $e^{4x/(6A+3x)}$, is lower than $\frac{A+x}{A}$, for any $x \in (0, A]$, then $\tilde{\theta} > 0$. Hence, Part (a) is proved.

We now consider the case: $\theta'(\Delta) < \theta \leq \theta''(\Delta)$. A few calculations show that the restriction $\theta \leq \theta''(\Delta)$, implies: $V_1(T_2^* - \Delta, T_2^*) \geq V_2(T_2^* - \Delta, T_2^*)$. Hence, the preemption argument sketched above applies again and the equilibrium is $\{\max\{\underline{T}_1, t_0\},\, T_2^*\}$ (refer to Figure 1, panel (b)). Notice that, at $\Delta = \Delta'$, $\theta'(\Delta') = \theta''(\Delta')(= \tilde{\theta})$.Therefore, this case applies only when $\Delta \geq \Delta'$. This completes the proof of Part (b).

Notice that the restriction $\theta > \max\{\theta'(\Delta),\, \theta''(\Delta)\}$, implies that $\theta > \theta'(\Delta)$ and that $\theta > \theta''(\Delta)$, simultaneously. In its turn, $\theta > \theta'(\Delta)$ implies that $T_1^* > T_2^* - \Delta$ and hence that $V_1(t_1, T_2^*)$ is increasing in the whole interval $t_1 \in [t_0, T_2^* - \Delta]$. The restriction $\theta > \theta''(\Delta)$ implies $V_2(T_2^* - \Delta, T_2^*) > V_1(T_2^* - \Delta, T_2^*)$. Hence, it is in each firm's interest to wait until $T_2^* - \Delta$, while the preemption argument does not apply. This completes the proof of Part (c).

Finally, we annalize the case: $\tilde{\theta} < \theta \leq \theta'(\Delta)$. The restriction $\theta \leq \theta'(\Delta)$ implies $T_1^* \leq T_2^* - \Delta$. However, if $\theta > \tilde{\theta}$, as shown in Part (a), $V_1(T_1^*, T_2^*) < V_2(T_1^*, T_2^*)$. Therefore, by Assumption 5, the first firm becomes the leader and invest at T_1^*, while the second invests at T_2^*. This completes the proof of Part (d).

Proof of Proposition 2

As a preliminary, notice, that $\bar{T} > T_2^* - \Delta$ for any $\theta \in [0, 1]$.

Notice moreover that some tedious calculations grant us that: $\theta'''(\Delta) \geq \theta''(\Delta)$.

As before, we start characterizing the optimal strategy for the follower.

When $t_1 \geq T_2^* - \Delta$, the innovation follower will never wait more than Δ, simply because $t_1 \geq T_2^* - \Delta$. Hence, his available strategies are:

(1) wait exactly Δ periods to grasp the benefit of the spillover;

(2) invest immediately after the innovation leader and

(3) wait for a time span shorter than Δ (to exploit the exogenous technological

externality) and then invest (therefore, without exploiting the inter-firm spillover).

First we compare what the innovation follower obtains by waiting Δ periods (strategy 1) with what he gets by investing immediately after the innovation leader (strategy 2). Hence, we determine when $V_2(t_1, t_1 + \Delta) \geq V_2(t_1, t_1)$. This inequality immediately boils down to:

$$\begin{aligned} & \frac{4Ax}{9br} e^{-r(t_1+\Delta-t_0)} - (1-\theta)\gamma x e^{-(r+\rho)(t_1+\Delta-t_0)} \\ \geq \quad & \frac{4Ax}{9br} e^{-r(t_1-t_0)} - \gamma x e^{-(r+\rho)(t_1-t_0)}, \end{aligned}$$

which, in its turn, is satisfied when: $t_1 \leq \bar{T}$. Hence, the innovation follower never chooses to immediately follow the leader for any $t_1 \in [T_2^* - \Delta, \bar{T})$.

Next, we compare strategy 1 with strategy 3.

In doing so, it is necessary to distinguish the case: $\bar{T} \geq T_2'$ from the case $\bar{T} < T_2'$. Notice that the inequality $\bar{T} \geq T_2'$ is satisfied when $\theta \geq 1 - \frac{r+\rho}{r} e^{\rho\Delta} + \frac{\rho}{r} e^{(r+\rho)\Delta}$, and some calculations allow us to verify that: $\theta''(\Delta) \geq 1 - \frac{r+\rho}{r} e^{\rho\Delta} + \frac{\rho}{r} e^{(r+\rho)\Delta}$.

Hence, in cases (a) and (b), $\bar{T} \geq T_2'$.

Suppose now that the leader invests at $t_1 \in (T_2^* - \Delta, T_2' - \Delta]$. In this interval, the payoff function for a follower who does not exploit the inter-firm spillover is always increasing. In fact, this function is concave with a global maximum at $t_2 = T_2'$ *for any* t_1(refer to the Proof for Proposition 1). Hence, it is optimal for the follower to invest later than $T_2' - \Delta$, which implies that the spillover is actually exploited.

When $t_1 \in (T_2' - \Delta, T_2']$, the optimal strategy for the innovation follower must be determined by comparing what it gets by delaying its investment for Δ periods with what can be obtained by investing at T_2'. Hence, we need to determine when $V_2(t_1, t_1 + \Delta) - V_2(t_1, T_2') \geq 0$. This inequality immediately boils down to:

$$\begin{aligned} & \frac{4Ax}{9br} \left[e^{-r(t_1+\Delta-t_0)} - e^{-r(T_2'-t_0)} \right] \\ & -\gamma x \left[(1-\theta) e^{-(r+\rho)(t_1+\Delta-t_0)} - e^{-(r+\rho)(T_2'-t_0)} \right] \geq 0. \end{aligned} \tag{A.4}$$

It is easy to show that the left hand side of (A.4) is non-increasing in t_1 in the whole interval $(T_2' - \Delta, T_2']$. Evaluate equation (A.4) at $t_1 = T_2'$, and–

exploiting equation (A.1)–substitute out T_2' when convenient, to obtain:

$$e^{-r(T_2'-t_0)}\frac{4Ax}{9br}\left[e^{-r\Delta}-1-r(1-\theta)\frac{e^{-(r+\rho)\Delta}}{r+\rho}+\frac{r}{r+\rho}\right]\geq 0,$$

which is fulfilled when $\theta \geq 1-\frac{r+\rho}{r}e^{\rho\Delta}+\frac{\rho}{r}e^{(r+\rho)\Delta}$. Hence, under this restriction, the follower's strategy of waiting Δ periods is chosen for any $t_1 \in (T_2'-\Delta, T_2']$.

Finally, strategy 3 can never be optimal for $t_1 \in (T_2', \bar{T}]$ just because the payoff function for a follower who does not exploit the spillover is decreasing in $t_2 \in (t_1, \bar{T}]$ and thus there is no point in waiting when the leader has already invested; recall moreover that the immediate investment strategy has already been proven to be dominated by a time Δ delay.

Hence, in cases (a) and (b) the follower's optimal reply to the innovation leader's decision to invest is to wait exactly Δ periods to grasp the inter-firm spillover and then invest.

The analysis for case (c) must be splitted into two sub-cases.

c1) When $\theta \in [1-\frac{r+\rho}{r}e^{\rho\Delta}+\frac{\rho}{r}e^{(r+\rho)\Delta}, \theta''(\Delta))$, then $\bar{T} \geq T_2'$ and the analysis developed above applies.

c2) When $\theta \in [0, 1-\frac{r+\rho}{r}e^{\rho\Delta}+\frac{\rho}{r}e^{(r+\rho)\Delta})$, then $\bar{T} < T_2'$. Notice, however, that it is possible to prove that $T_2'-\Delta < \bar{T}$. In the time interval $t_1 \in (T_2^*-\Delta, T_2'-\Delta]$ the optimal strategy is again to wait Δ and exploit the inter-firm spillover, because the follower's payoff function $V_2(t_1,t_2)$ is increasing in $t_2 \in (t_1, T_2'-\Delta]$.

When $t_1 \in (T_2'-\Delta, \bar{T}]$, the optimal strategy for the innovation follower must be determined by comparing what he gets by delaying his investment for Δ periods with what can be obtained by investing at T_2'. Unfortunately, it is not possible to characterize analytically the sub-intervals in which the two alternative strategies prevail. Let us denote by $\breve{T}_1 \in (T_2'-\Delta, \bar{T}]$ the instant when $V_2(t_1, t_1+\Delta) = V_2(t_1, T_2')$. $\breve{T}_1 \in (T_2'-\Delta, \bar{T}]$ because: $V_2(t_1, t_1+\Delta) - V_2(t_1, T_2')$ is non-increasing in t_1; $\lim_{\epsilon\to 0}[V_2(T_2'-\Delta+\epsilon, T_2'+\epsilon) - V_2(T_2'-\Delta+\epsilon, T_2')] > 0$ and $V_2(\bar{T}, \bar{T}+\Delta) - V_2(\bar{T}, T_2') < 0$, in fact, by definition $V_2(\bar{T}, \bar{T}+\Delta) = V_2(\bar{T}, \bar{T})$ and $V_2(\bar{T}, \bar{T}) < V_2(\bar{T}, T_2')$. Hence, for $t_1 \in (T_2'-\Delta, \breve{T}_1]$ strategy i) is optimal, while for $t_1 \in (\breve{T}_1, T_2']$ the innovation follower decides to innovate at T_2' (strategy 3).

We conclude our characterization for the follower's optimal strategy by noting that $V_2(t_1, t_1+\Delta)$ is a concave function with its maximum at the right of

$T_2^* - \Delta$.

We now proceed to analyzing the first firm's behavior.

If $\theta \in (\theta'''(\Delta), 1]$ (case a), the first innovator is aware of the fact that her competitor will always invest with a delay of Δ periods. Hence, she computes her payoff for $t_1 \in [T_2^* - \Delta, \bar{T}]$ which is:

$$V_1(t_1, t_1 + \Delta) = \frac{A^2}{9br} +$$
$$\left\{ \left[\frac{4(A+x)x}{9br} - \gamma x e^{-\rho(t_1 - t_0)} \right] - \frac{(2A+3x)x}{9br} e^{-r\Delta} \right\} e^{-r(t_1 - t_0)}.$$

Notice that $\theta \geq \theta''(\Delta)$, implies that $V_2(T_2^* - \Delta, T_2^*) \geq V_1(T_2^* - \Delta, T_2^*)$; notice moreover that the payoff function $V_1(t_1, t_1 + \Delta)$ reaches its maximum at $t_1 = \hat{T}_1$.

Consider now the equation: $V_1(t_1, t_1 + \Delta) = V_2(t_1, t_1 + \Delta)$, which has a unique solution: $T_1^{ip} = t_0 - \frac{1}{\rho} \ln \left(\frac{3(2A+x)(1-e^{-r\Delta})}{9br\gamma[1-(1-\theta)e^{-(r+\rho)\Delta}]} \right)$.Notice that $T_1^{ip} > \hat{T}_1$ if $\theta > \theta'''(\Delta)$, hence $\theta \in (\theta'''(\Delta), 1]$, $T_1^{ip} \in [\hat{T}_1, \bar{T}]$. (This happens because the inter-firm spillover is very high, implying a high payoff for the second firm). Because the $V_2(t_1, t_1 + \Delta)$ curve lies above the $V_1(t_1, t_1 + \Delta)$ curve for $t_1 \in [T_2^* - \Delta, \hat{T}_1]$, no firm has an incentive to preempt its rival before $\hat{T}_1$. The first firm, under Assumption 5, has no incentive to delay her adoption beyond $\hat{T}_1$, because $V_1(t_1, t_1 + \Delta)$ is decreasing for $t_1 \in [\hat{T}_1, \bar{T}]$. Hence, in the interval $[T_2^* - \Delta, \bar{T}]$, the unique sub-game perfect equilibrium is $t_1 = \hat{T}_1$, $t_2 = \hat{T}_1 + \Delta$. The first panel in Figure 3 portrays the situation analyzed here. This proves Part (a).

In case (b) ($\theta \in [\theta''(\Delta), \theta'''(\Delta)]$), the first innovator is aware, again, of the fact that her competitor will invest with a delay of Δ periods. In this case, however, $T_1^{ip} \in [T_2^* - \Delta, \hat{T}_1]$: in fact, with respect to the previous case, the reduction in the spillover parameter shifts downward the follower's payoff function (as depicted in Figure 3, panel (b)) and, for $t_1 \in [T_1^{ip}, \bar{T}]$ the $V_2(t_1, t_1 + \Delta)$ curve lies below the $V_1(t_1, t_1 + \Delta)$ curve. Hence, $t_1 = T_1^{ip}$ is part of the unique pure strategy equilibrium, due to the usual preemption argument. This proves Part (b).

When considering the follower's optimal strategy in case (c), we must distinguish the two sub-cases: c1) $\theta \in [1 - \frac{r+\rho}{r} e^{\rho\Delta} + \frac{\rho}{r} e^{(r+\rho)\Delta}, \theta''(\Delta))$, and c2) $\theta \in [0, 1 - \frac{r+\rho}{r} e^{\rho\Delta} + \frac{\rho}{r} e^{(r+\rho)\Delta})$.

c1) When $\theta \in [1 - \frac{r+\rho}{r} e^{\rho\Delta} + \frac{\rho}{r} e^{(r+\rho)\Delta},\ \theta''(\Delta))$, the first innovator is aware of the fact that her competitor will invest with a delay of Δ periods. In this case one can show that the unique solution for the equation $V_1(t_1, t_1+\Delta) = V_2(t_1, t_1+\Delta)$, lies outside the interval $[T_2^* - \Delta, \bar{T}]$. (i.e. $T_1^{ip} < T_2^* - \Delta$). Because the follower's payoff is lower than the first innovator's one, in the whole interval $[T_2^* - \Delta, \bar{T}]$, the usual preemption argument moves the equilibrium back to the region considered in Proposition 1. This case is portrayed in panel (c) of Figure 3.

c2) When $\theta \in [0, 1 - \frac{r+\rho}{r} e^{\rho\Delta} + \frac{\rho}{r} e^{(r+\rho)\Delta})$, the first firm is aware that, if she chooses $t_1 \in [T_2^* - \Delta, \breve{T}_1]$, the follower picks an innovation time characterized by a delay of span Δ, while if she chooses $t_1 \in [\breve{T}_1, \bar{T}]$, the follower innovates at time $T_2' > \bar{T}$.

Suppose first that the follower innovates with a delay of Δ. In this case, because $T_1^{ip} < T_2^* - \Delta$, it is obvious that $V_1(t_1, t_1 + \Delta) > V_2(t_1, t_1 + \Delta)$ for any $t_1 \in [T_2^* - \Delta, \bar{T}]$ and hence, a fortiori, for any $t_1 \in [T_2^* - \Delta, \breve{T}_1]$. Suppose now that the follower innovates at time T_2'. In this case, again, $V_1(t_1, T_2') > V_2(t_1, T_2')$ for $t_1 \in [T_2' - \Delta, \bar{T}]$ and hence, a fortiori, for any $t_1 \in [\breve{T}_1, \bar{T}]$. (To show this, consider that $V_1(T_2', T_2') = V_2(T_2', T_2')$ and that $\partial[V_1(t_1, T_2') - V_2(t_1, T_2')]/\partial t_1 < 0$.). Hence, in the whole interval $[T_2^* - \Delta, \bar{T}]$, the follower's payoff is lower than the first innovator's one and the usual preemption argument moves the equilibrium back to the region considered in Proposition 1. This proves Part (c).

Proof of Proposition 3.

As usual, we start characterizing the optimal strategy for the follower.

The proof of Proposition 2 implies that the innovation follower will never wait Δ, for any $t_1 \geq \bar{T}$. Hence, his available strategies are:

(1) invest immediately after the innovation leader, and

(2) wait for a time span shorter than Δ (to exploit the exogenous technological externality) and then invest without exploiting the inter-firm spillover.

The proof of Proposition 1 implies that - when the innovation follower decides to wait, he invests at T_2', for any $t_1 \in (T_2' - \Delta, T_2']$. In fact, the payoff function for the follower, $V_2(t_1, t_2)$ has a maximum at T_2'.

In the proof for Proposition 2, we showed that, for $\theta \in [1 - \frac{r+\rho}{r} e^{\rho\Delta} + \frac{\rho}{r} e^{(r+\rho)\Delta}, 1]$, then $\bar{T} \geq T_2'$. Hence, under this parameter restriction, the second innovator invests immediately after the innovation leader. In fact, it is never in the follower's interest to wait Δ periods, exactly because $t_1 > \bar{T}$. Moreover the follower's payoff function is decreasing in t_2 in the whole interval $t_1 \in [\bar{T}, \infty)$. Hence, the follower sees no point in waiting.

In contrast, when $\theta \in [0, 1 - \frac{r+\rho}{r} e^{\rho\Delta} + \frac{\rho}{r} e^{(r+\rho)\Delta})$, then $\bar{T} < T_2'$. Therefore, under this parameter restriction, the second innovator invests at T_2' when $t_1 \in [\bar{T}, T_2']$; when $t_1 \in (T_2', \infty)$, the innovation follower invests immediately after the innovation leader because its payoff function is decreasing in t_2.

We now analyze the first firm's behavior.

Suppose first, that $\theta \in [1 - \frac{r+\rho}{r} e^{\rho\Delta} + \frac{\rho}{r} e^{(r+\rho)\Delta}, 1]$, so that the first firm knows that - as soon as she innovates - the rival firm immediately sinks the innovation cost.

Hence, the payoff for the first firm is:

$$V_1(t_1, t_1) = \int_{t_0}^{t_1} \pi_1^{00} e^{-r(t-t_0)} dt + \\ + \int_{t_1}^{\infty} \pi_1^{11} e^{-r(t-t_0)} dt - \gamma x e^{-r(t_1 - t_0)}. \tag{A.5}$$

Maximization of (A.5) with respect to t_1 under the constraint $t_1 \geq \bar{T}$ yields that the first firm optimal timing is:

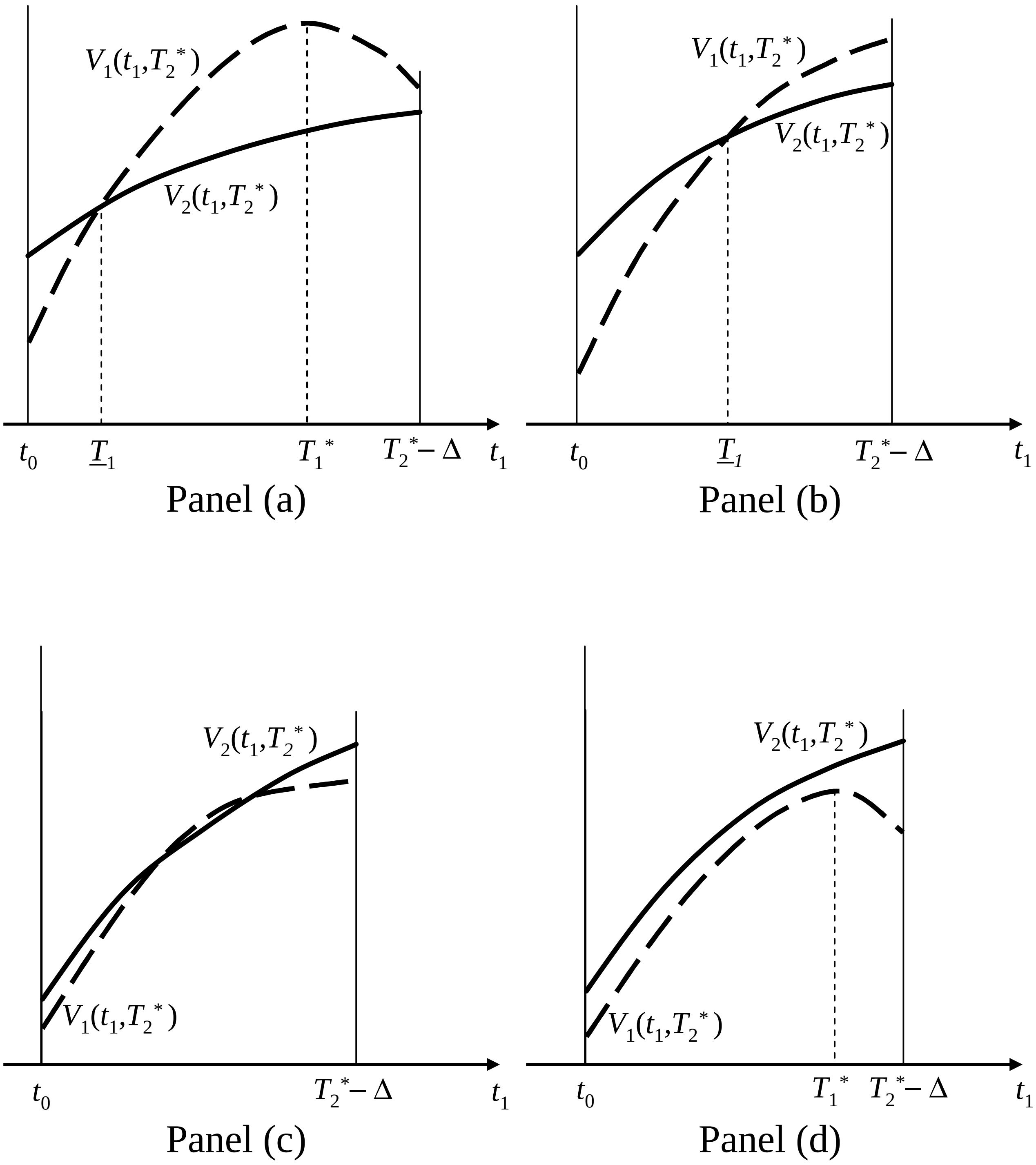
$\bar{T}$ if $\theta \geq \hat{\theta}(\Delta) = 1 - e^{(r+\rho)\Delta} \left[1 - (r+\rho) \frac{1 - e^{-r\Delta}}{r} \frac{4A}{2A+x} \right]$,

$T^{le} = t_0 - \frac{1}{\rho} \ln \left(\frac{2A+x}{9b\gamma(r+\rho)} \right)$ if $\theta < \hat{\theta}(\Delta)$.

It is now easy to show that $\hat{\theta}(\Delta) \geq 1 - \frac{r+\rho}{r} e^{\rho\Delta} + \frac{\rho}{r} e^{(r+\rho)\Delta}$. Accordingly, when $\theta \geq \hat{\theta}(\Delta)$, both firms invest at: $\bar{T}$. This proves part (a).

To prove part (b), notice first that – when $\theta \in [1 - \frac{r+\rho}{r} e^{\rho\Delta} + \frac{\rho}{r} e^{(r+\rho)\Delta}, \hat{\theta}(\Delta))$– the second firm invests immediately after the first firm. Hence, the equilibrium is T^{le}.

Finally, when $\theta \in [0, 1 - \frac{r+\rho}{r} e^{\rho\Delta} + \frac{\rho}{r} e^{(r+\rho)\Delta})$, the second firm invests at T_2' when $t_1 \in [\bar{T}, T_2']$; when $t_1 \in (T_2', \infty)$, the second firm invests immediately after the first. In the interval $t_1 \in [\bar{T}, T_2')$ no equilibrium exists: in fact $V_1(t_1, T_2') > V_2(t_1, T_2')$ for $t_1 \in [\bar{T}, T_2')$. This can be shown by noting that $V_1(T_2', T_2') = V_2(T_2', T_2')$, and that $\frac{\partial V_1(t_1, T_2')}{\partial t_1} \leq \frac{\partial V_2(t_1, T_2')}{\partial t_1}$ for $t_1 \in [T_2' - \Delta, T_2')$, and therefore, a fortiori, for $t_1 \in [\bar{T}, T_2')$. Because in this interval, the second firm's payoff is lower than the first firm's one, the usual preemption argument moves the equilibrium back to the region considered in Proposition 2. In the interval $t_1 \in [T_2', \infty)$, the equilibrium is T^{le}, (which is always larger than T_2'): the first firm, being aware that the second immediately follows her innovation decision, maximizes (A.5). Hence, in the interval $t_1 \in [\bar{T}, \infty)$, either the equilibrium is T^{le} (when $V_1(\bar{T}, T_2') \geq V_1(T^{le})$) or it does not exist (when $V_1(\bar{T}, T_2') < V_1(T^{le})$). This proves part (c).



Panel (a)

Panel (b)

Panel (c)

Panel (d)

Figure 1: Alternative behaviors for the firms’ discounted payoffs in the early interval

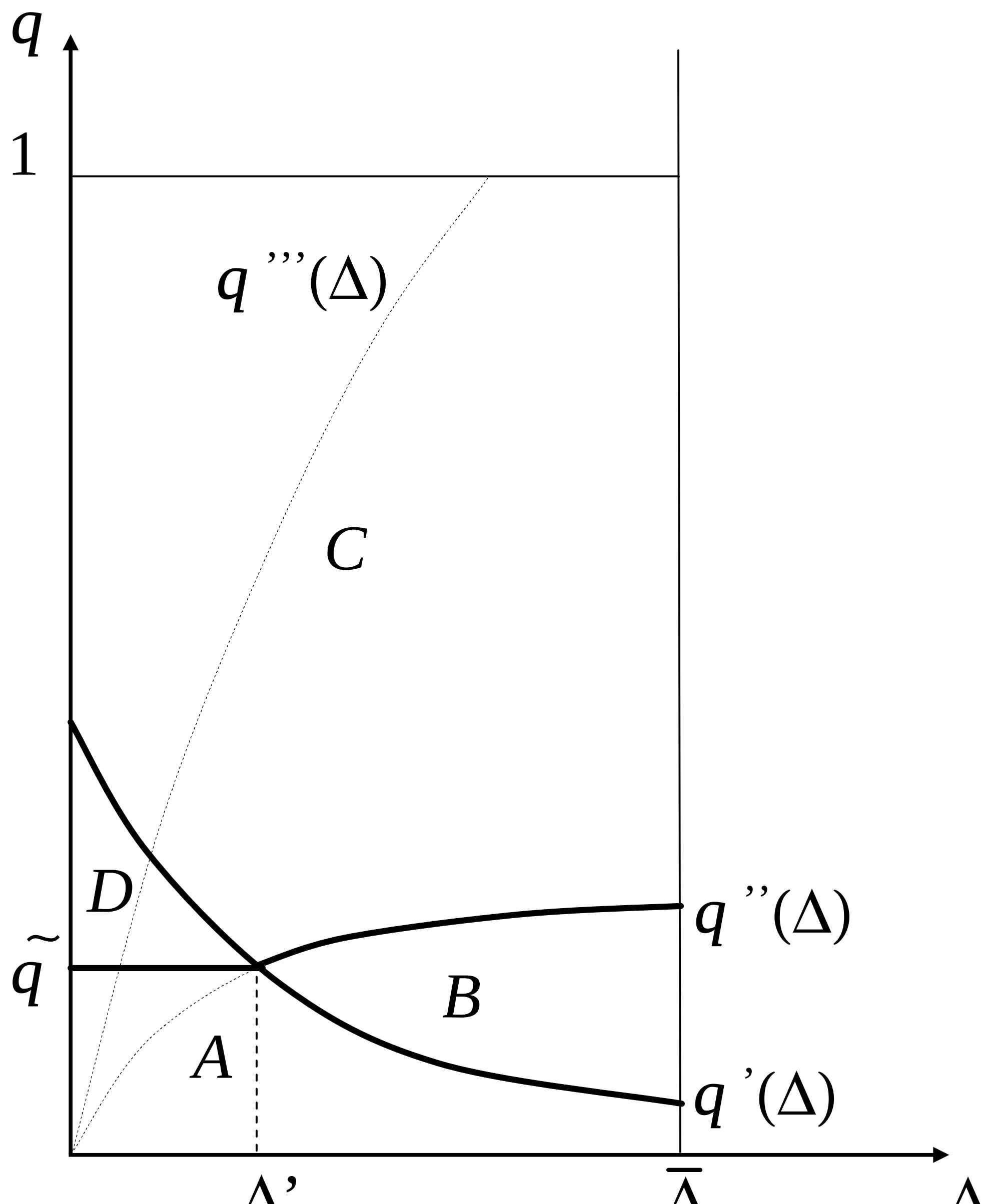


Figure 2. Parameter sets leading to alternative behaviors of the payoffs in the early and intermediate intervals

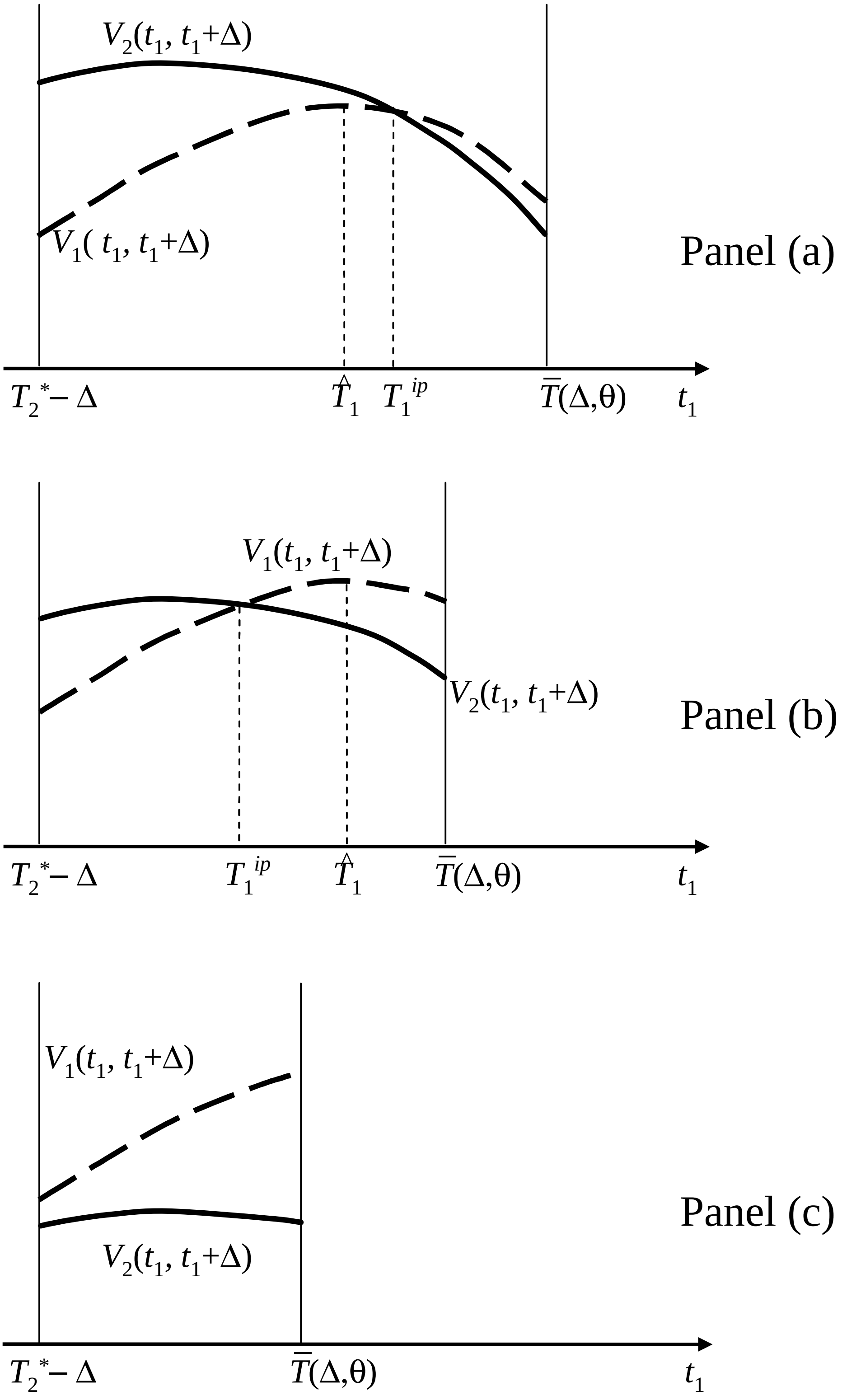
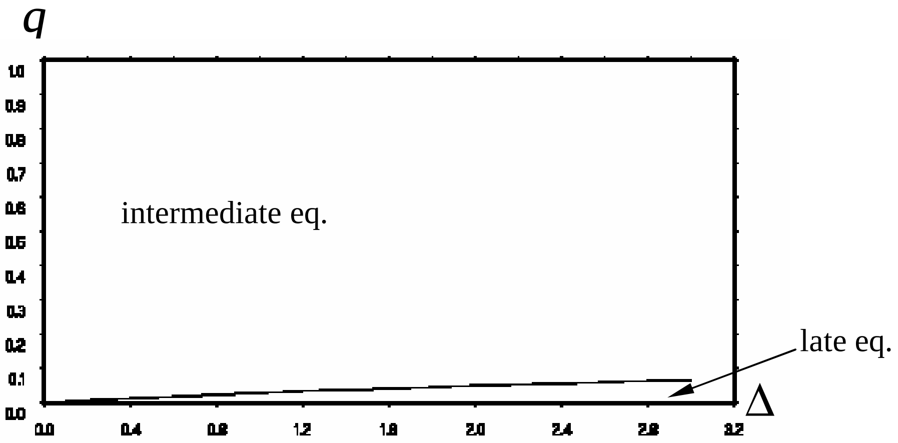
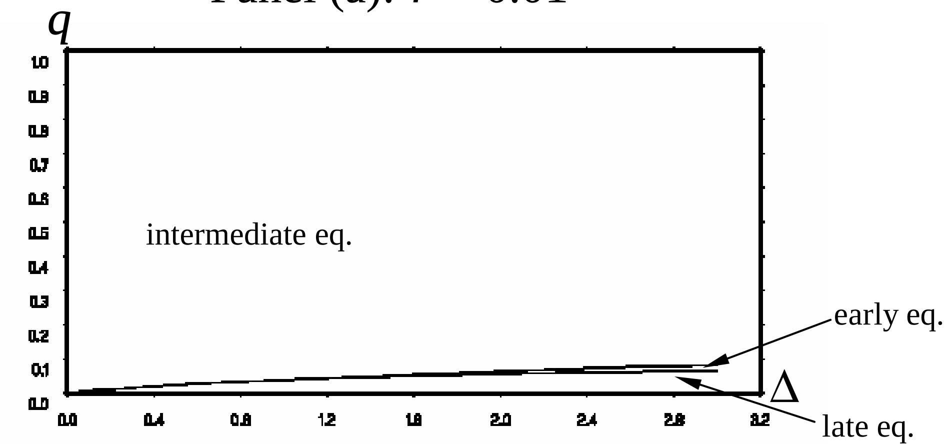


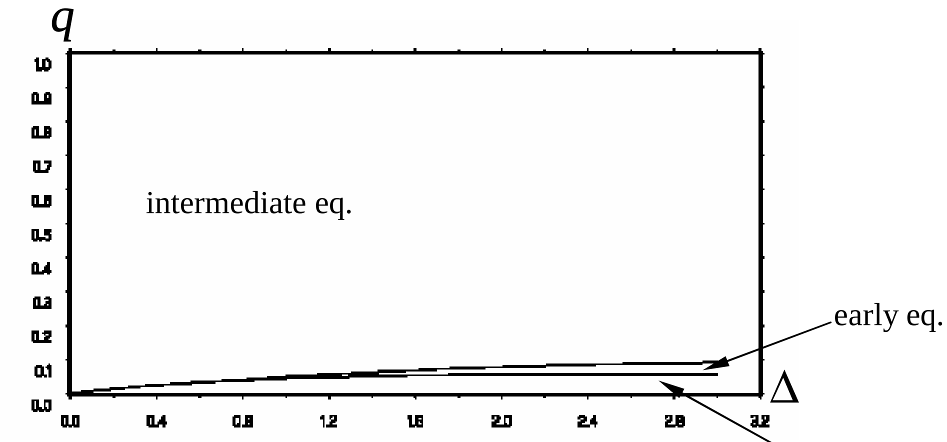
Figure 3: Alternative behaviors for the firms’ discounted payoffs in the intermediate interval



Panel (a): $r = 0.01$

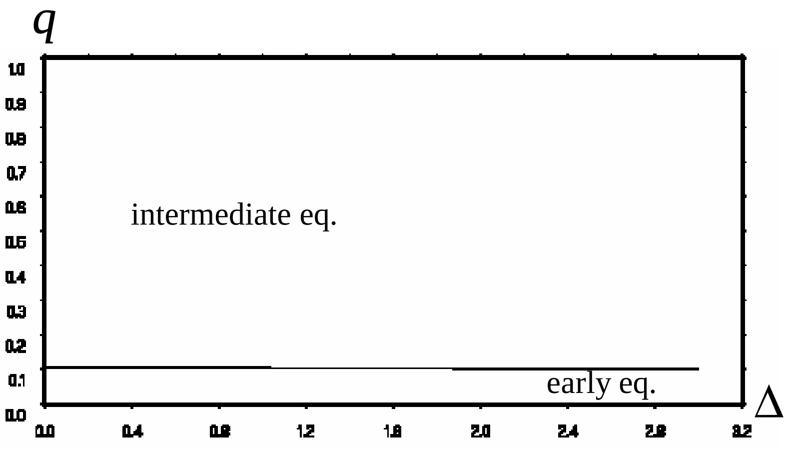


Panel (b): $r = 0.05$

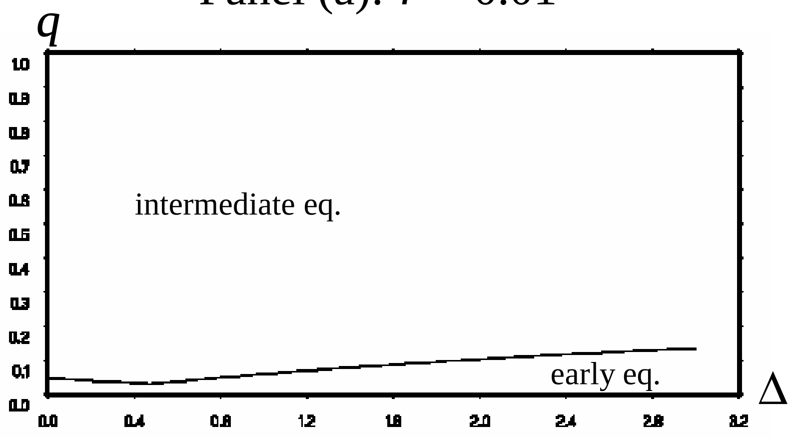


Panel (c): $r = 0.09$

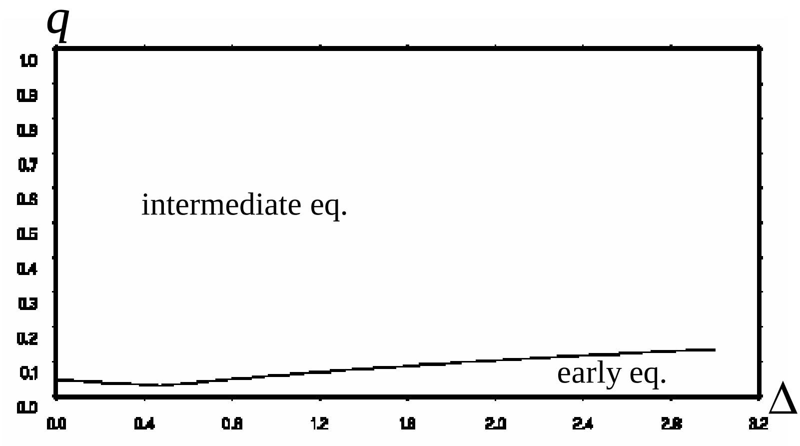
Figure 4 Equilibrium selection – minor innovation ($x = 0.05$)



Panel (a): r = 0.01

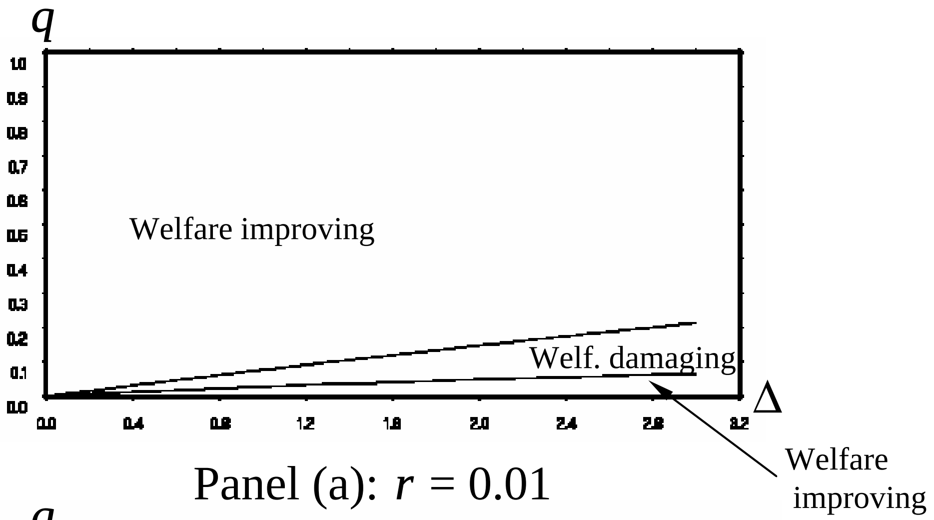


Panel (b): r = 0.05

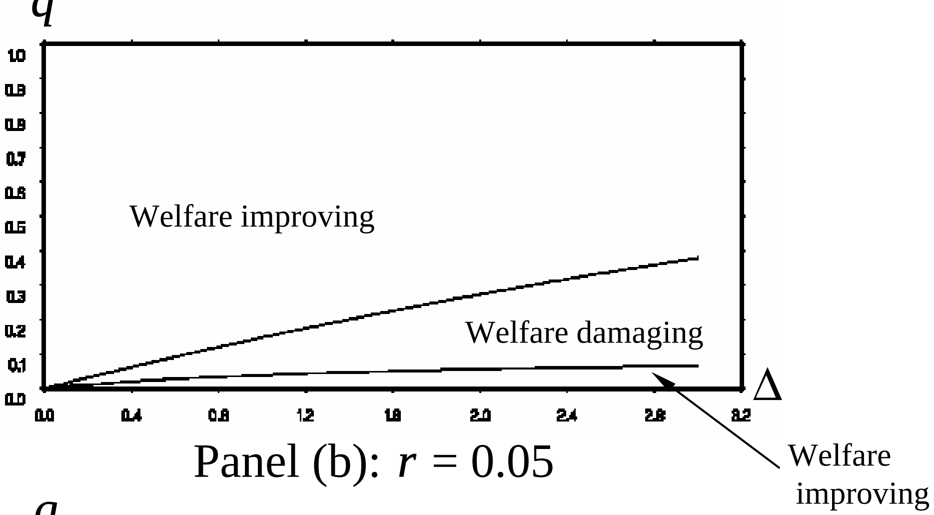


Panel (c): r = 0.09

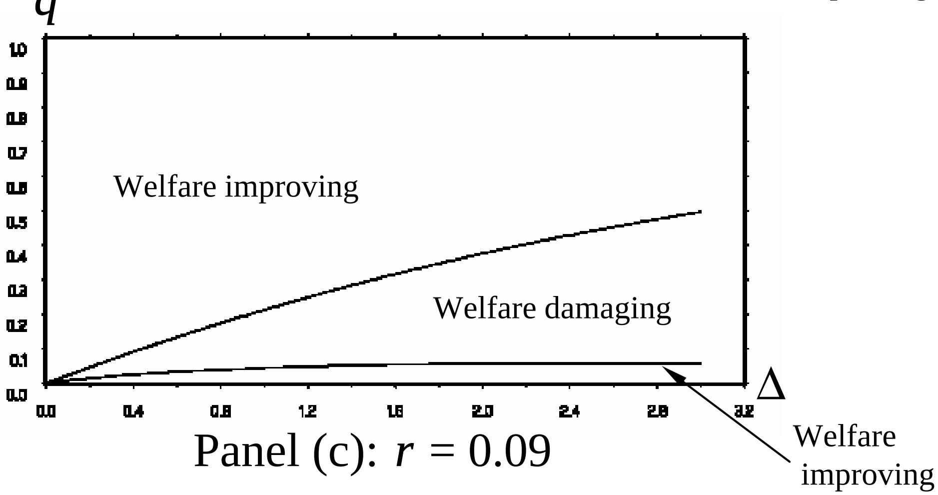
Figure 5 Equilibrium selection – major innovation (x = 0.50)



Panel (a): $r = 0.01$

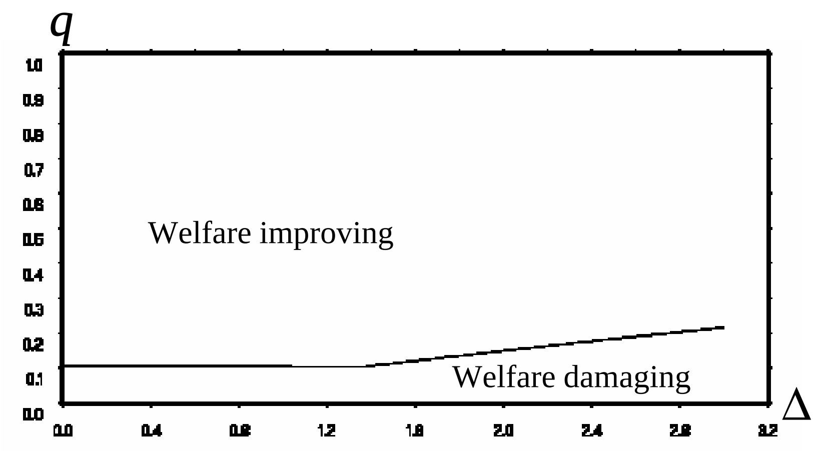


Panel (b): $r = 0.05$

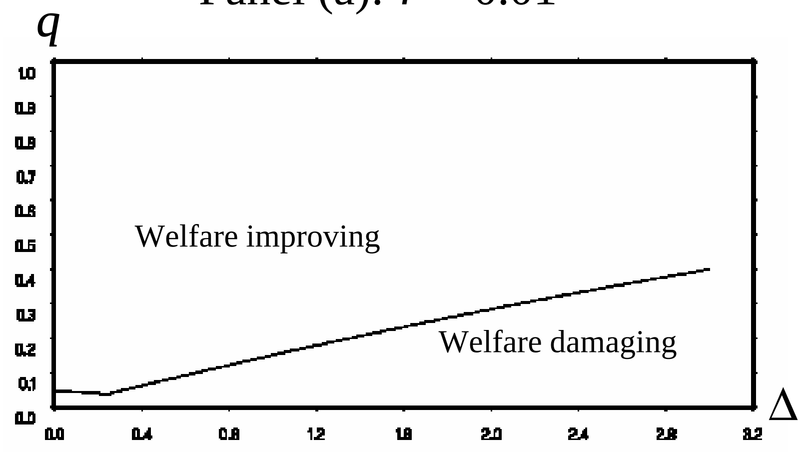


Panel (c): $r = 0.09$

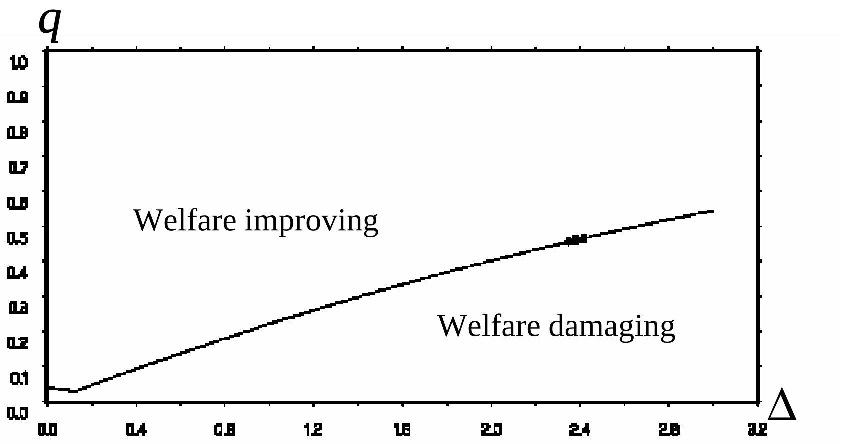
Figure 6 Welfare improving RJV ($x = 0.05$)



Panel (a): r = 0.01



Panel (b): r = 0.05



Panel (c): r = 0.09

Figure 7 Welfare improving RJV (x = 0.50)