Spillovers, disclosure lags, and incentives to innovate: Do oligopolies over-invest in R&D?

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Abstract

We develop a dynamic duopoly, where firms have to take into account a technological externality, that reduces over time their innovation costs, and an inter-firm spillover, that lowers only the second comer's R&D cost. This spillover exerts its effect after a disclosure lag. We identify three possible equilibria, which are classified, according to the timing of R&D investments, as early, intermediate, and late. The intermediate equilibrium is subgame perfect for a wide parameters range. When the innovation size is large, it implies that the duopolistic market equilibrium involves underinvestment. Hence, even in presence of a moderate degree of inter-firms spillover, the competitive equilibrium calls for public policies aimed at increasing the research activity. When we focus on minor innovations – the case in which, according to the earlier literature, the market equilibrium underinvests – our results imply that the policies aimed at stimulating R&D have to be less sizeable than suggested before, despite the presence of an inter-firm spillover.

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1 Introduction

Understanding firms' decision to innovate is of fundamental importance to design policies aimed at maximizing welfare. The firms' decisions are driven by their incentives; hence the market structure in which firms operate plays a crucial role in determining the pace of technical progress. This provides a strong motivation for the analysis of oligopolies, which are the most widespread market configuration.

In our study, we analyze a duopoly in which – as in many recent contributions (e.g. Stenbacka and Tombak (1994), Hoppe (2000), and Schmidt-Dengler (2006)) – the R&D cost shrinks over time thanks to general advances in knowledge and technology. In addition to this standard technological externality, firms take into account a spillover that lowers the second comer's innovation cost. A distinctive features of our duopoly game is that the spillover exerts its effect after a time period which we label "disclosure lag".

When the oligopolistic competition is driven by the innovative activity, inter-firm spillovers play an important role. In fact, they alter the length of the follower's strategic delay, and hence the leader's cost advantage period.[1] Also the presence of a disclosure lag has important strategic implications. In fact, the first innovator is aware that the second comer cannot exploit the spillovers before the relevant information are disclosed. Hence, the behavior of the interacting firms is influenced by the existence of a production cost-advantage period for the first comer.

What we find is that in our framework three types of equilibria arise, while the existing contributions, following Fudenberg and Tirole (1985), identifies two possible market equilibria: an early and a late one.

This literature, which starts with Reinganum (1981), and is excellently surveyed by Hoppe (2002), identifies two driving forces characterizing the equilibria: the length of the follower's strategic delay, and the intensity of the competitive pressure. In the early equilibrium, the second innovator delays his decision to invest for a relatively long period. This choice is driven by the desire to grasp the benefit of technical progress, that reduces the innovation cost as time goes by. The follower's optimal choice implies a long competitive advantage period for the innovator leader, which favors the latter's payoff at the expenses of the former's one. Hence, to avoid being preempted, the first mover invests "very soon", and the R&D investment is socially excessive. The preemption possibility also implies rent equalization.

[1] The importance of spillovers for R&D is underscored in De Bondt (1996), who provides many reference to earlier contributions, which, however, adopt static even if multi-stage frameworks.

In contrast, a late equilibrium arises once technical progress has substantially reduced the innovation costs, so that an innovation leader cannot emerge, because the rival would immediately copy her decision. In this case, any innovator – anticipating that there will be no leadership – waits until her choice maximizes the joint discounted stream of net profits. The collusive flavour of this equilibrium is apparent: accordingly, Fudenberg and Tirole's analysis implies that this type of market equilibrium underinvests. Their contribution also suggests that the early equilibrium is subgame perfect when the size of the innovation is large. In this case, in fact, the per-period first innovator profits are considerable, which triggers the preemptive behavior.

We label "intermediate" the third type of equilibrium we identify, since the decisions to innovate take place, for both firms, at dates positioned between the early and the late ones. This happens because the first innovator knows that the second comer will exploit the spillovers as soon as the relevant information is obtained, i.e. exactly at the end of the disclosure lag. Because in the early equilibrium the follower waits more than the disclosure lag, the leader's competitive advantage period is shorter in the intermediate equilibrium than in the early one. This harms the leader's discounted profits, but the follower is benefited. Therefore the competitive pressure − that leads to rent equalization−is weaker than in the early equilibrium, and does not force the leader to invest "very soon". However, the competitive pressure is high enough to avoid a late equilibrium.

We select the equilibrium for the overall game by applying the subgame perfection criterion, and we find that the intermediate equilibrium is particularly relevant because it is the prevailing one for a large range of the parameters set. Notice that – in our framework – the natural indicators of an highly competitive environment, namely an equilibrium with R&D diffusion and rent equalization, do not imply that the R&D investment is excessive from the social planner's perspective.

When the innovation size is large, the intermediate equilibrium implies that the duopolistic market equilibrium involves underinvestment. An underinvesting equilibrium in presence of a major innovation is a result that contrasts not only with the literature following Fudenberg and Tirole, but also with the previous contributions inspired by Loury (1979), and by Lee and Wilde (1980).[2] The relevant implication is that, according to our model,

[2] Loury, and Lee and Wilde assume that a new technique becomes suddenly available, and immediately triggers the industry investment in R&D. The competitive pressure induced by the market structure implies that the equilibrium involves an R&D investment that is higher than the social optimum. This result can partially be ascribed to the tourna-

the competitive equilibrium calls for public policies aimed at increasing the research activity.

When, instead, we focus on minor innovations, the equilibrium we describe is more realistic than the late one, which is characterized by simultaneous adoptions, a phenomenon seldom observed in the real world. Notice that, with minor innovations – the case in which, according to the earlier literature, the market equilibrium underinvests – the prevalence of the intermediate equilibrium imply that the policies aimed at stimulating R&D have to be less sizeable than suggested before, despite the presence of an inter-firm spillover.

To understand why the intermediate equilibrium is subgame perfect, consider first the case of an innovation of limited size. In this situation, when the spillover is (relatively) high, the follower grasps (relatively) large benefits from investing at the end of the disclosure lag, so that he finds optimal to select this strategy for a long time interval. This makes the leader unwilling to wait until the late equilibrium prevails, which gives rise to the intermediate equilibrium. When, instead, the spillover is very low, the subgame perfect equilibrium is the late one, because the "immediate reply" strategy for the follower becomes optimal at earlier dates.[3]

In contrast, when the innovation size is large, an early equilibrium may emerge, because a major innovation, bringing a large cost advantage to the leader, enhances her incentive to be first. However, the higher the spillover, the sooner the second comer – due to the reduction in innovation costs – optimally invests in reply to an early leader's investment. This reduces the leaders' efficiency advantage period, leading to the dominance of the intermediate equilibrium. Moreover, a (relatively) high spillover increases the second comer's payoff in the intermediate time interval, and this softens the leader's preemption incentive to invest. This milder competition implies higher payoffs for both firms in the intermediate equilibrium.

These results, being driven by the assumption of an inter-firm spillover coupled with the one of a disclosure lag, differ from the ones already obtained in the literature. In fact, Riordan (1992) focuses on the early equilibrium, and analyses the impact of price and entry regulations on the timing of

ment nature of these models. In a non-tournament model, Beath *et al* (1989) underscore the role of the competitive threat as a major determinant of R&D expenditure. Because the larger is the competitive threat, the more resources firms invest in R&D, overinvestment is more likely, the larger is the size of the innovation. Delbono and Denicolò (1991), again in a non-tournament framework, find that the equilibrium R&D effort can be lower than the social optimum if the marginal cost of the innovation is low.

[3]As in the previous literature, a small cost reduction, implying a weak incentive to innovate first, does not give rise to an equilibrium with preemption.

adoption. Because these regulatory schemes tend to reduce the first innovator's rents, they are likely to delay the early adoption, which can be socially beneficial.

Stenbacka and Tomback (1994) analyze the role of experience, which implies that the probability of successful implementation of an innovation is an increasing function of the time distance from the investment date. As for welfare, they show that a collusive adoption timing may improve welfare when compared with the market equilibrium. This happens when the pace of technical progress is fairly high: when this is the case, a collusive adoption is beneficial because the industry can fully take advantage of the reduction of innovation cost. In contrast, a competitive market equilibrium, being driven by the incentives to obtain a strategic advantage, induces a premature adoption.

In Hoppe (2000), firms are uncertain about the profitability of the innovation. Her framework differs from the one by Fudenberg and Tirole, thanks to the presence of technological uncertainty, which induces an asymmetry between the leader and the follower. The latter observes the leader's outcome, and hence becomes aware about the actual profitability characterizing the new technique. This informational spillover may bring about a second-mover advantage. Moreover, an high probability of failure induces a late simultaneous adoption because it curtails the first mover expected payoff. When the late equilibrium is subgame perfect, Hoppe finds that an earlier simultaneous adoption would be welfare increasing, while the result are less definite when the early equilibrium prevails.

Weeds (2002) presents a tournament version of Fudenberg and Tirole (1985), in which profits evolve stochastically. She suggests that the early (late) equilibrium over(under)-invests; however the late equilibrium is closer to the social optimum than the early one.[4]

The paper proceeds in the standard way. In Section 2 we present our model. In Section 3 we discuss the equilibrium concept adopted in the analysis and we compute the different market equilibria, in which firms compete both in the innovation and in the product stages. Then, subgame

[4] The presence of an inter-firm spillover assimilates our model to the frameworks proposed by Katz and Shapiro (1987), Dutta *et al.* (1995), and Hoppe and Lehmann-Gruber (2005) among others. Katz and Shapiro introduce an extreme form of technological spillover, assuming that, in a duopoly, the follower can adopt at no cost the new technology as soon as the leader has invested. This hypothesis induces the possibility of a second mover advantage. A similar approach is followed in Dasgupta (1988). Dutta *et al.* demonstrate that the second mover advantage may prevail as subgame perfect equilibrium output in product innovation games. Hoppe and Lehmann-Gruber generalize the previous results by analyzing the issue of multiple peaks in the leader's payoff function.

perfectness is invoked as a selection device among market equilibria. In Section 4 we spell out the welfare implications of our analysis. Concluding comments in Section 5 end the paper.

2 The model

2.1 The production stage and its welfare implications

We consider an industry composed of two firms, 1 and 2, which, in each (infinitesimally short) period, are involved in a two–stage interaction: first they decide whether to innovate or not, and then they compete *à la* Cournot. Time is continuous and firms' horizon is infinite. Firms discount future profits at the common rate r. Market demand is linear and equal to: $P = a - bQ$, where P is the market clearing price and $Q = q_1 + q_2$ is the total quantity supplied. Each firm has a unit cost of production c.

The R&D cost evolves over time.[5] In each period t firm i ($i = 1, 2$) decides whether to invest in R&D or not. This investment immediately yields a cost-reducing process innovation, which shrinks the unit production cost by an amount x, with $x < c$. Hence firm i's post–innovation production cost is $C(q_i) = (c - x)q_i$.

Each firm's payoff will depend not only on its adoption date but also on its rival's one. If both firms have not invested up to period t, their individual profits in the Cournot subgame at t are those of the pre–innovation stage, i.e.

$$\pi_i^{00} = \frac{A^2}{9b}, \tag{1}$$

where $A = a - c$. The superscript $\{00\}$ indicates that both firms do not innovate at t. The instantaneous welfare (computed *à la Marshall*) is then equal to:

$$W^{00} = \frac{4}{9}\frac{A^2}{b}. \tag{2}$$

If instead only one firm, say firm 1, invests in R&D at t, it benefits of an efficiency advantage, and obtains a higher market share. The market price at t decreases in comparison with the pre-innovation level, while the individual profits become:

[5] The functional forms and dynamics for the firms' R&D costs are modeled in Section 2.2.

$$\pi_1^{10} = \frac{(A+2x)^2}{9b}; \pi_2^{10} = \frac{(A-x)^2}{9b}, \tag{3}$$

where $\{10\}$ indicates that firm 1 has invested in R&D while firm 2 has not. Notice that $\pi_1^{10} > \pi_2^{10}$, $\pi_1^{10} > \pi_1^{00}$ and $\pi_2^{10} < \pi_2^{00}$. Because $q_2^{10} = \frac{A-x}{3b}$, to preserve the duopolistic structure characterizing our market we need to introduce:

Assumption 1: $A > x$.

This hypothesis implies that, in a Cournot environment, the cost-reducing innovation is non−drastic (see, Denicolò (1996)). In case of asymmetric behavior at t, welfare is:

$$W^{10} = \frac{8A(A+x) + 11x^2}{18b}, \tag{4}$$

with $W^{10} > W^{00}$.

Finally, we need to compute the outcomes when both firms have innovated at t. In this case, being more efficient, they both produce more than in the *status quo*; therefore, the market price is lower. Individual profits at t are:

$$\pi_i^{11} = \frac{(A+x)^2}{9b}, \tag{5}$$

where the superscript $\{11\}$ indicates that both firms have innovated.

Obviously, $\pi_1^{10} > \pi_1^{11}$; notice, moreover, that the difference between π_1^{10} and π_1^{11} is increasing in x: when only one firm enjoys a cost advantage, she obtains a larger market share while benefiting from an higher price to cost margin.

When both firms have innovated, the social welfare is:

$$W^{11} = \frac{4(A+x)^2}{9b}, \tag{6}$$

with $W^{11} > W^{10}$ (by Assumption 1).

When firms simultaneously invest in R&D, individual profits rise from (1) to (5) and welfare jumps from (2) to (6). Alternatively, firms may behave asymmetrically, so that there are both an innovation leader and a follower. Under these circumstances individual profits first change from to (1) to (3) (and welfare from (2) to (4)) and then from (3) to (5) (and welfare from (4) to (6)).

2.2 R&D costs

In our set-up, firms decides whether to invest in a fixed-size research project. For the first firm investing in R&D, the innovation cost evolves over time according to the following equation:

$$C_1(t_1) = \gamma x e^{-\rho(t_1 - t_0)}, \quad \text{for} \quad t_1 \in [t_0, \infty), \tag{7}$$

where t_1 is the calendar time when the first firm has introduced the innovation. Hence, we are assuming that the innovation becomes technically feasible at time t_0 at a cost, γx, which then decreases at the constant rate $\rho \geq 0$, thanks to the advances in pure research and to the availability of new results obtained in related fields. Of course, this form of technical progress is exogenous to any single firm. It is clear from (7) that, if a firm innovates in period t, R&D costs are sunk at that time.

As for the second firm introducing the innovation, the cost evolution is described by the following equation:

$$C_2(t_2) = \begin{cases} \gamma x e^{-\rho(t_2 - t_0)} & \text{for} \quad t_2 \in [t_1, t_1 + \Delta) \\ (1-\theta)\gamma x e^{-\rho(t_2 - t_0)} & \text{for} \quad t_2 \in [t_1 + \Delta, \infty) \end{cases}, \tag{8}$$

where $\theta \in [0, \bar{\theta}]$ is the inter-firm spillover parameter; $\bar{\theta}$ shall be assumed as being strictly lower than unity. In fact, with $\theta = 1$, the follower – bearing no innovation cost – would always invest at the end of the disclosure lag. Hence, this (irrealistic) particular case would deliver trivial results. Δ is the delay needed to grasp the benefit stemming from the rival's innovative activity. Hence, Δ is the exogenously determined disclosure lag.

Whenever $\theta > 0$ the innovation is only partially appropriable: the second comer enjoys a reduction in R&D costs by imitating his competitor at $t_2 \geq t_1 + \Delta$. In our formulation, it takes time to imitate an innovation: in his classic study, Mansfield (1985) reports that in 59% of cases it takes more than twelve months to the innovator's rival to obtain the relevant information. More recently, Cohen *et al.* (2002) compute that the average adoption lag for unpatented process innovation is 2.03 and 3.37 years in Japan and in the US, respectively. An obvious but important consequence of our assumption is that the introduction of an innovation grants to the leader a cost advantage (and hence higher profits) for a time period (at least) equal to Δ.

We stylize an extremely simple form of spillover: it would have been preferable to consider a stochastic disclosure lag, with a probability of information diffusion depending upon the time elapsed from the introduction of the innovation, and on the follower's imitation effort. The latter should be used to influence also the spillover size.[6] However, even the simplest stochastic formulation – namely the one involving a constant probability of information diffusion coupled with a fixed spillover size – precludes the attainment of explicit results.[7] Hence, our formulation has been chosen as the optimal compromise between analytical tractability and "realism".

In what follows we will restrict the values for θ, Δ and γ. In particular, we now introduce the following technical assumptions:

Assumption 2: $\max\left\{\frac{x}{A+x}, 1-\frac{4A}{6A+3x}\left(1+\frac{r}{\rho}\frac{2A+3x}{6A+3x}\right)^{\frac{\rho}{r}}\right\} \leq \bar{\theta} < 1.$

Assumption 3: $\Delta \leq \bar{\Delta} = \frac{1}{r}\ln\left(1+\frac{r}{\rho}\frac{2A+3x}{6A+3x}\right)$;

Assumption 4: $\gamma \geq \bar{\gamma} = \frac{4A\exp(\rho\bar{\Delta})}{9b(r+\rho)(1-\bar{\theta})}$.

Assumption 2 allows for sufficiently high spill-over levels, which makes the discussion more interesting.

The purpose of Assumption 3 is to limit the number of cases that we need to consider. To verify that Assumption 3 does not restrict Δ to values too short to be sensible, we compute $\bar{\Delta}$ when x approaches 0 (since this choice lowers $\bar{\Delta}$), the annual interest rate is 0.03, and $\rho = \{0.01,\ 0.05,\ 0.09\}$.[8] With these values, $\bar{\Delta}$ becomes, respectively, equal to $\{23.105,\ 6.077,\ 3.512\}$. Hence, the restriction implied by Assumption 3 is realistic in most contexts. From the vantage point of economic analysis, a low Δ is interesting, because it makes more relevant the role of the inter-firms spillovers.

Assumption 4 guarantees the existence of all the three types of equilibria in the space $[0,\bar{\Delta}]\ x\ [0,\bar{\theta}]$.

3 The market equilibria

In this Section we discuss the equilibria in the non-cooperative R&D game. To this purpose, we first explain the equilibrium concept adopted to solve the model. Because the payoff functions, in general, are not single-peaked, we deal with the existence of multiple equilibria. We divide time in three sub-intervals, in such a way that in each interval the equilibrium is unique.

[6] To endogenize θ we could have followed Jin and Troege (2006), which suggest that firms can raise it, paying a convex imitation cost. Nevertheless, we preferred not to pursue this development of the model, because our framework is already fairly complex: any further extension requires a much heavier use of numerical techniques to determine and select the equilibrium. For the same reason we do not endogenize the lenght of the disclosure lag.

[7] Notice also that a constant probability of information disclosure does not represent an improvement upon our formulation, since the sparse empirical evidence available suggests that the probability of successful imitation increases over time.

[8] These values for ρ have a relevant economic interpretation that will become apparent later.

We then select the globally unique equilibrium referring to the concept of subgame perfectness.

3.1 The equilibrium concept

As already mentioned, in our set-up only one research project is available to the firms: hence, the choice to innovate at time t_i is an irreversible stopping decision. Therefore, our model belongs to the class of symmetric timing games, which can be divided into two sub-classes, depending upon which firm (the one that moves first or the one that moves second) obtains the higher payoff.

We can make this point more precise, by assuming for the moment that we have exogenously assigned the task of moving first to one of the two firms. In this case, there is a first mover advantage if the firm that must move first obtains the higher payoff. If, instead, the first mover obtains the lower payoff, there is a second mover advantage. Obviously the first mover is assumed to behave optimally, choosing the innovation time that maximizes its payoff, given the second mover optimal choice.

To deal with first mover advantage games, we drop the hypothesis of exogenously assigned roles and we follow Hoppe and Lehman-Gruber (2005) assuming that:

Assumption 5: If the two firms are indifferent between being the first or the second mover at any date t, then the role of the leader is played by the firm with a female CEO[9] and the role of the follower is played by the other firm, which is run by a male CEO.

Assumption 5 is used to rule out, as it happens in most of the literature, the possibility of coordination failures as an equilibrium outcome. In other words, firms do not choose to move at the same instant of time if they know that they would regret this choice afterwards.[10]

The logic to obtain the unique subgame perfect equilibrium in first-mover advantage games can be described by exploiting Panel (a) in Figure 1. Denoting by T_2^* the second comer's optimal timing, the payoff function $V_1(t_1, T_2^*)$ gives firm's 1 net profits when she invests at time t_1, while the rival invests at T_2^*; these profits are discounted back to t_0 for convenience. $V_2(t_1, T_2^*)$ gives firm's 2 discounted payoff when he invests at T_2^*, while the first invests at t_1. Because $V_1(t_1, T_2^*)$ is single-peaked at $t_1 = T_1^*$, the leader

[9]Say firm 1, which will henceforth be referred to as if it were a female.

[10]From a technical standpoint, – as Hoppe and Lehman-Gruber (2005) remark – an equilibrium involving coordination failures cannot be obtained in the case of a continuous-time game without a grid, in which equilibria are defined to be the limits of discrete-time mixed strategies (Fudenberg and Tirole, (1985) and (1991)).

would like to adopt first at T_1^*. But the roles of innovation leader and follower are not pre-assigned. Hence, when the second firm knows that the other will adopt at time T_1^*, it is in his interest to preempt at time $T_1^* - dt$. By backward induction, we conclude that the equilibrium strategy for the first innovator is to invest as soon as the leader's payoff is equal to the follower's one (i.e. at $\underline{T}_1$). (Assumption 5 grants us that the first innovator is actually firm 1.) Notice that the preemption argument spelled out above yields equal payoffs to the two firms in the subgame perfect equilibrium. Hence, in this case the equilibrium involves rent dissipation.

[Figure 1 about here]

In dealing with second mover advantage games, we rely again on Hoppe and Lehman-Gruber's analysis. In this case, they assume that the equilibrium is driven by expectations and make the following hypothesis:

Assumption 6: Whenever the innovation leader payoff is lower than the second comer's, firm 1 believes that firm 2 never enters first.

The logic to obtain the unique subgame perfect equilibrium in this case can be explained by means of Panel (d) in Figure 1. $V_1(t_1, T_2^*)$ is single-peaked at $t_1 = T_1^*$, moreover, the $V_1(t_1, T_2^*)$ curve lies below the $V_2(t_1, T_2^*)$ curve for any $t_1 \leq T_1^*$. Hence firm 1 chooses $t_1 = T_1^*$ (the date granting her the highest possible payoff) while no firm has an incentive to preempt its rival before date t_1.

Assumption 6 (and therefore the equilibrium it implies) may seem arbitrary. In fact, it rules out the mixed-strategies equilibria, often referred to as a war of attrition (Fudenberg and Tirole (1991)). However−if we reject Assumption 6−our firms would start to randomize at T_1^*, obtaining, in every instant of time an expected payoff equal to the leader's one. Hence, the rejection of Assumption 6 leads – in the second mover advantage cases – to the attainment of equilibria implying later adoption dates but the same expected payoff than the one we study. In what follows, it will become apparent that removing Assumption 6 is harmless for our results.

3.2 Alternative market equilibria

In the next Sub-sections, we divide the time line $[t_0, \infty)$ into three sub-intervals, in which three different equilibria arise.

When the innovation leader decides to invest "very early", the follower's optimal strategy is to wait more than Δ periods before imitating the leader. This gives rise to an early equilibrium, which will be analyzed in Sub-section 3.2.1.

We then consider the equilibrium that arises when the innovation leader delays her innovation, so that the follower's optimal choice is to invest exactly Δ periods after the leader, grasping the inter-firm spillover as soon as possible. We shall refer to this situation as the intermediate equilibrium, which will be analyzed in Sub-section 3.2.2.

Finally, the innovation leader may decide to invest "very late". In this case, the R&D cost is so low that it is optimal for the second firm to immediately enter upon the rival's investment, forsaking the inter-firm spillover. An equilibrium with these characteristics is labeled the late one and it will be discussed in Sub-section 3.2.3.

We denote by $V_1(t_1, t_2)$ the discounted stream of future profits obtained by the first firm investing at t_1 while her rival sinks the innovation cost at t_2, that is:

$$V_1(t_1,t_2) = \int_{t_0}^{t_1} \pi_1^{00} e^{-r(t-t_0)} dt + \int_{t_1}^{t_2} \pi_1^{10} e^{-r(t-t_0)} dt + \\ + \int_{t_2}^{\infty} \pi_1^{11} e^{-r(t-t_0)} dt - C_1(t_1) e^{-r(t_1-t_0)}. \tag{9}$$

Accordingly, the second firm's payoff is:

$$V_2(t_1,t_2) = \int_{t_0}^{t_1} \pi_2^{00} e^{-r(t-t_0)} dt + \int_{t_1}^{t_2} \pi_2^{10} e^{-r(t-t_0)} dt \\ + \int_{t_2}^{\infty} \pi_2^{11} e^{-r(t-t_0)} dt - C_2(t_2) e^{-r(t_2-t_0)}. \tag{10}$$

3.2.1 The early equilibrium

By investing early, the leader incurs a high innovation cost (equation (7)), because pure research has not yet provided many results upon which to build upon. The high innovation cost is the reason why the follower prefers to invest with a delay longer than Δ years: in fact, if he waits more than Δ, he not only nets the benefits from imitation, but he can also grasp relevant additional gains from pure research, which is still producing results that are quantitatively important for reducing the R&D cost.

Maximizing (10) with respect to t_2, we obtain the follower's optimal choice, which is to invests at

$$T_2^* = t_0 - \frac{1}{\rho} \ln \left(\frac{4A}{9b\gamma(r+\rho)(1-\theta)} \right). \tag{11}$$

This solution applies when the leader sinks the costs at $t_1 \leq T_2^* - \Delta$.[11]

The comparative statics on T_2^* gives sensible results. In particular, the higher the inter-firms spillover, the sooner the second comer invests: a high θ reduces – *ceteris paribus* – the follower's costs and therefore anticipates his investment date.[12]

Having qualified the follower's optimal investment timing, we analyze the leader's behavior.

As a preliminary, we define by T_1^* the value for t_1 that maximizes $V_1(t_1, T_2^*)$, i.e.

$$T_1^* = t_0 - \frac{1}{\rho} \ln \left(\frac{4(A+x)}{9b\gamma(r+\rho)} \right), \tag{12}$$

and we introduce three thresholds for θ that – as it will be proved in the Appendix – are crucial to formalize the leader's behavior.[13]

The first threshold, $\theta'(\Delta)$, is the value such that, for $\theta \in [0, \theta'(\Delta)]$, we have that $T_1^* < T_2^* - \Delta$. In words, when θ is below $\theta'(\Delta)$, the maximal payoff for the leader, $V_1(T_1^*, T_2^*)$, is attained in the interval $[t_0, T_2^* - \Delta)$. This happens because a low θ allows for the typical inverted-U leader's payoff function. In fact, this shape is determined by two opposing forces. An increase in the leader's adoption time induces a reduction in her innovation cost, which increases $V_1(t_1, T_2^*)$, but implies also a shortening in her efficiency advantage period, which reduces $V_1(t_1, T_2^*)$. When t_1 is relatively low, the former effect dominates the latter because the cost reduction induced by the technological externality is quantitatively relevant. When θ is low, T_2^* is large: a low θ, implying an high follower's costs, postpones his optimal investment date, which gives room for the second effect to prevail.

The second relevant threshold, $\theta''(\Delta)$, is such that, for $\theta \in [0, \theta''(\Delta)]$, then $V_1(T_2^* - \Delta, T_2^*) \geq V_2(T_2^* - \Delta, T_2^*)$.

Finally, $\tilde{\theta}$, is the value such that, for $\theta \in [0, \tilde{\theta})$, $V_1(T_1^*, T_2^*) \geq V_2(T_1^*, T_2^*)$: an high inter-firm spillover – favouring the follower – may lead to situations

[11] Assumptions 3 and 4 guarantee that $T_2^* - \Delta \geq t_0$ for any $\Delta \in [0, \bar{\Delta}]$, $\theta \in [0, \bar{\theta}]$.

[12] An increase in A or a decrease in b induce an expansion in per-period profit and hence they anticipate the second comer's decision to innovate; an increase in γ or in r delays his investment decision, because the innovation is more costly, or the future profits are more heavily discounted. The technical progress parameter ρ plays a twofold role: on the one hand, its increase implies that, at any date t_2, the innovation costs are lower, which calls for an earlier investment; on the other hand, a faster reduction in innovation costs may induce a firm to wait because it knows that the cost will quickly become smaller. With a low spillover, the first direct effect prevails over the second indirect one; in contrast, when θ is high, the impact of an increase in ρ on T_2^* may well be positive for realistic parameter values.

[13] Assumptions 1, 2, and 4 guarantee that $T_1^* \geq t_0$ for any $\Delta \in [0, \bar{\Delta}]$, $\theta \in [0, \bar{\theta}]$.

where even the highest possible leader's payoff is lower than the follower's one.[14]

It is now important to distinguish first from second mover's advantage situations.

Our timing game is of the first mover advantage type in two cases. First, it belongs to this sub-class when the leader's payoff function $V_1(t_1, T_2^*)$ has an inverted-U shape, and we have that $V_1(T_1^*, T_2^*) > V_2(T_1^*, T_2^*)$. The second case arises when $V_1(t_1, T_2^*)$ and $V_2(t_1, T_2^*)$ are increasing in $[t_0, T_2^* - \Delta]$, and $V_1(T_2^* - \Delta, T_2^*) \geq V_2(T_2^* - \Delta, T_2^*)$.

The first case applies when $\theta \leq \min\{\tilde{\theta}, \theta'(\Delta)\}$, i.e. in area A in Figure 2. There, the leader's payoff function displays the inverted-U shape because $\theta \leq \theta'(\Delta)$, and we have that $V_1(T_2^* - \Delta, T_2^*) \geq V_2(T_2^* - \Delta, T_2^*)$, because $\theta \leq \tilde{\theta}$. Figure 1, panel (a), portrays this case, for which the equilibrium is obtained applying the preemption argument; hence, we conclude that the equilibrium strategy for the first innovator is to invest as soon as the leader's payoff is equal to the follower's one (at $\underline{T}_1 \leq T_1^*$ in panel (a)).

[Figure 2 about here]

The second sub-case is represented in Figure 1, panel (b), which highlights a preemptive equilibrium at $\{\underline{T}_1, T_2^*\}$. In this case, the inter-firm spillover parameter θ is above the threshold $\theta'(\Delta)$, so that the first innovator's payoff does not reach an internal maximum in the interval $[t_0, T_2^* - \Delta]$, but it is below the threshold $\theta''(\Delta)$, which implies $V_1(T_2^* - \Delta, T_2^*) \geq V_2(T_2^* - \Delta, T_2^*)$. Hence, when $\theta \in (\theta'(\Delta), \theta''(\Delta)]$, i.e. in area B in Figure 2, there is a first mover advantage in some left interval of $T_2^* - \Delta$, and the preemption argument applies.

We now need to discuss the two cases in which our timing game is of the second mover advantage type. In the first instance, $V_1(t_1, T_2^*)$ and $V_2(t_1, T_2^*)$ are increasing but $V_1(T_2^* - \Delta, T_2^*) < V_2(T_2^* - \Delta, T_2^*)$. In the second case, the leader's payoff function $V_1(t_1, T_2^*)$ has an inverted-U shape, but $V_1(T_1^*, T_2^*) < V_2(T_1^*, T_2^*)$.

Figure 1, panel (c), depicts the first second mover advantage sub-case. Here, we have $\theta \in (\max\{\theta'(\Delta), \theta''(\Delta)\}, \bar{\theta}]$, (area C in Figure 2) so that the first innovator's payoff does not reach a maximum in the interval $[t_0, T_2^* - \Delta]$, and $V_1(T_2^* - \Delta, T_2^*) < V_2(T_2^* - \Delta, T_2^*)$. In words, θ is sufficiently high that

[14] In the Appendix we show that:
$\theta'(\Delta) \equiv 1 - \frac{A}{A+x} e^{\rho\Delta}$,
$\theta''(\Delta) \equiv 1 - \frac{4Are^{(r+\rho)\Delta}}{(r+\rho)(6A+3x)(e^{r\Delta}-1)+4Ar}$, and that
$\tilde{\theta} \equiv 1 - \frac{A}{A+x}\left[1 + \frac{4xr}{\rho 3(2A+x)+r(2A-x)}\right]^{\frac{\rho}{r}}$.

even the optimal timing for the first mover yields her a payoff that is lower than the follower's one. Note that Assumption 6 implies that the first firm's equilibrium adoption date is $T_2^* - \Delta$.

Finally, Figure 1, panel (d) portrays the case in which θ is between $\tilde{\theta}$ and $\theta'(\Delta)$: the spillover is such that the first adopter payoff function reaches a maximum in $[t_0, T_2^* - \Delta]$, (because $\theta \leq \theta'(\Delta)$), but its maximum is below the corresponding follower's payoff (because $\theta > \tilde{\theta}$). In this case, being Δ small, the leader's payoff function is inverted U-shaped even if the spillover parameter is relatively high. However, θ is high enough to guarantee that, even at T_1^*, the first mover enjoys a payoff that is lower than the follower's one. Again, Assumption 6 implies that the first firm's equilibrium adoption date is T_1^*. In Figure 2, the Area where this case applies is D.

The above arguments are formally presented in:

Proposition 1

When Assumptions 2, 3 and 4 are satisfied, for $t_1 \in [t_0, T_2^* - \Delta]$,

(a) if $\theta \in [0, \min\{\tilde{\theta},\ \theta'(\Delta)\}]$ the unique subgame perfect equilibrium is $\{\max\{\underline{T}_1, t_0\}, T_2^*\}$, where $\underline{T}_1$ is the earliest adoption date for the first firm, such that, $V_1(\underline{T}_1, T_2^*) \geq V_2(\underline{T}_1, T_2^*)$;

(b) if $\theta \in (\theta'(\Delta),\ \theta''(\Delta)]$, the unique subgame perfect equilibrium, is $\{\max\{\underline{T}_1, t_0\}, T_2^*\}$;

(c) if $\theta \in (\max\{\theta'(\Delta),\ \theta''(\Delta)\}, \bar{\theta}]$, the unique subgame perfect equilibrium is $\{T_2^* - \Delta, T_2^*\}$;

(d) if $\theta \in (\tilde{\theta},\ \theta'(\Delta)]$, the unique subgame perfect equilibrium is $\{T_1^*, T_2^*\}$.

Proof: See the Appendix.

3.2.2 The intermediate equilibrium

We now analyze what happens when the innovation leader invests after $T_2^* - \Delta$.

When $t_1 > T_2^* - \Delta$, the follower's choice is among to copy immediately, to wait less than Δ, and to wait exactly Δ before investing (to grasp the inter-firm spillover).[15] We define $\bar{T}$ as the first date such that the second firm payoff gained by the "immediately following" strategy, becomes as high as the payoff granted by the decision of waiting Δ periods before investing in R&D. When $t_1 < \bar{T}$, the follower's optimal choice is to wait exactly Δ

[15] Waiting more than Δ can never be optimal for the follower, just because such strategy calls for an investment at T_2^* as a reply to a previous leader's investment.

periods before innovating. In fact, the innovation cost is still high enough that it is convenient for the follower to let the spillover reduce his R&D costs even if this choice grants to the leader an efficiency advantage for Δ periods.

Again, whether the game is of the first or of the second mover advantage type depends on the values taken by Δ and by the spillover parameter θ.

A high θ (i.e. $\theta \in (\theta'''(\Delta), \bar{\theta}])$[16], grants to the follower a second mover advantage for a large sub-interval of $(T_2^* - \Delta, \bar{T}]$. In fact, if t_1 is close to $T_2^* - \Delta$, the leader bears an high innovation cost. Hence, a large spillover induces a second mover advantage because its relevant size more than compensates for the first innovator's efficiency advantage. Only when t_1 is not too far from $\bar{T}$, the technological externality has already made the innovation cost rather low: this weakens the role of the inter-firms spillover, leading to a first mover advantage. This case is portrayed in Figure 3, Panel (a). By Assumption 6 the unique equilibrium is $\{\hat{T}_1, \hat{T}_1 + \Delta\}$, with $\hat{T}_1$ being the maximum for $V_1(t_1, t_1 + \Delta)$ in the interval $t_1 \in [T_2^* - \Delta, \bar{T}]$.

[Figure 3 about here]

When $\theta \in (\theta''(\Delta), \theta'''(\Delta)]$, the spillover parameter θ is sufficiently low to guarantee the existence of a first mover advantage in a large portion of $(T_2^* - \Delta, \bar{T}]$. This case is depicted in Figure 3, panel (b): in comparison with panel (a), the reduction in θ has shifted downward the follower's payoff function, inducing the existence of a preemption equilibrium at the intersection point T_1^{ip}. There, the payoffs for the two firms are identical. In fact, in this equilibrium the advantage in production costs, enjoyed by the leader for Δ periods, is exactly compensated by the lower R&D costs granted to the innovation follower by the joint effects of the inter-firm spillover and of the technological externality.

When $\theta \in [0, \theta''(\Delta)]$, the imitation benefit is small and the first firm payoff is always larger than the second firm's one. Panel (c) in Figure 3 depicts the behavior for the payoffs functions, which allows to conclude that in the interval $(T_2^* - \Delta, \bar{T}]$, the equilibrium is a mixed strategy one.[17] However, such a mixed strategy equilibrium, cannot be subgame perfect when we consider the whole interval $[t_0, \infty)$,[18] and therefore it is not relevant for our discussion.

[16] With $\theta'''(\Delta) = 1 - \frac{[4Ar+\rho(6A+3x)](e^{-r\Delta}-1)+r(2A+x)}{re^{-(r+\rho)\Delta}[4(A+x)-(2A+3x)e^{-r\Delta}]}$. A bit of algebra shows that $\theta'''(\Delta) > \theta''(\Delta)$, as depicted in Figure 2.

[17] In this equilibrium, firms start randomizing at $T_2^* - \Delta$.

[18] In fact any firm starting to randomize at $T_2^* - \Delta$ would be preempted in the earlier interval (consider Figure 1, panels (a) and (b), with Figure 3, panel (c), bearing in mind that the payoff functions are continuous at $T_2^* - \Delta$ and at T_2^*, respectively).

The above arguments are summarized in:

Proposition 2

Let $\bar{T} = t_0 - \frac{1}{\rho} \ln \left(\frac{4A(1-e^{-r\Delta})}{9br\gamma[1-(1-\theta)e^{-(r+\rho)\Delta}]} \right)$.

When Assumptions 2, 3 and 4 are satisfied, in the interval $t_1 \in (T_2^* - \Delta, \bar{T}]$,

(a) if $\theta \in (\theta'''(\Delta), \bar{\theta}]$, the unique subgame perfect equilibrium is $\{\hat{T}_1, \hat{T}_1 + \Delta\}$, where:

$$\hat{T}_1 = t_0 - \frac{1}{\rho} \ln \left(\frac{4(A+x) - (2A+3x)e^{-r\Delta}}{9b\gamma(r+\rho)} \right);$$

(b) if $\theta \in [\theta''(\Delta), \theta'''(\Delta)]$ the unique subgame perfect equilibrium is $\{T_1^{ip}, T_1^{ip} + \Delta\}$, where:

$$T_1^{ip} = t_0 - \frac{1}{\rho} \ln \left(\frac{3(2A+x)(1-e^{-r\Delta})}{9br\gamma[1-(1-\theta)e^{-(r+\rho)\Delta}]} \right). \qquad (13)$$

(c) if $\theta \in [0, \theta''(\Delta))$, there is no pure strategy equilibrium in the interval $[T_2^* - \Delta, \bar{T}]$.

Proof: See the Appendix.

$\bar{T}$ is raised by an increase in the inter-firm spillover: in fact, a more relevant benefit from imitation postpones the undertaking of a line of action that prescribes the forsaking of the benefit itself.[19]

More interestingly, we see from (13) that in case (b) an increase in the inter-firm spillover delays the equilibrium. This happens because the equilibrium $\{T_1^{ip}, T_1^{ip} + \Delta\}$ is preemptive: the first innovator sinks the R&D costs as soon as her payoffs becomes larger than the rival's one. Because an higher θ benefits the follower, it also softens the incentive to invest for the leader and hence mitigates the competitive pressure.

We conclude this Sub-section by jointly discussing the results obtained in Propositions 1 and 2, which allow us to select the equilibrium in the whole interval $[t_0, \bar{T}]$ for most parameters configurations.

When $\theta \in (\max\{\theta'(\Delta), \theta''(\Delta)\}, \bar{\theta}]$, which is in area C of Figure 2, the intermediate equilibrium is the one described in Proposition 2, parts (a) and (b)), while, in the interval $[t_0, T_2^* - \Delta]$, the equilibrium is $\{T_2^* - \Delta, T_2^*\}$ (Proposition 1, part (c)). Since $V_1(t_1, t_1 + \Delta)$ is increasing in the whole

[19] Apart from the effect of θ, the comparative static for $\bar{T}$ is quite similar to the one for T_2^*.

interval $t_1 \in (t_0, T_2^* - \Delta]$ (Figure 1, panel (c)), the intermediate equilibrium is the subgame perfect one in the interval $[t_0, \bar{T}]$: there is no first firm deviation payoff that can undermine this equilibrium for $t_1 \in [t_0, \bar{T}]$.

When $\theta \in [0, \theta''(\Delta)]$ (i.e. in the lower portion of area A and in area B of Figure 2), the intermediate equilibrium is not relevant: any firm investing in $[T_2^* - \Delta, \bar{T}]$ would be preempted in the earlier interval. (Bear in mind that the two payoff functions are continuous at $T_2^* - \Delta$ and at T_2^*, respectively, and consider Figure 1, panels (a) and (b), and Figure 3, panel (c)). Accordingly, the pure strategy preemption equilibrium which exists in $[t_0, T_2^* - \Delta]$ – as granted by Proposition 1, parts (a) and (b) – is the subgame perfect equilibrium in $[t_0, \bar{T}]$.

In the remainder of area A and in area D (i.e. for $\theta''(\Delta) < \theta < \theta'(\Delta)$), the first innovator payoff function has local maxima both in $(t_0, T_2^* - \Delta]$ and in $(T_2^* - \Delta, \bar{T}]$ (Proposition 1, parts (a) and (d) and Proposition 2, parts (a). Refer also to panels (a) and (b) in Figure 1 and to panel (a) in Figure 3). In this case, the equilibrium selection on the ground of the subgame perfectness criterion requires the use of numerical simulations. This analysis will be carried out in Sub-section 3.3.

3.2.3 The late equilibrium

Finally, if the innovation leader decides to invest "late" (i.e. when $t_1 \in [\bar{T}, \infty)$) the R&D cost is so low that it is optimal for the second firm to immediately enter upon rival's investment, without exploiting the inter-firm spillover.

In this case the first firm is aware that−as soon as she innovates−the second firm will "immediately" follow her decision, and invest. Hence, each firm takes her decision anticipating such a follower's behavior. This leads to an equilibrium where the two firms maximize their joint payoff: knowing that it will be immediately followed, each firm delays its innovation until its discounted sum of profits reaches its maximum. In this context, where firms remain symmetric, the maximization of a single firm's payoff coincides with their joint maximization.

Formally, the innovation leader's behavior is summarized by the following proposition.

Proposition 3

When Assumptions 2, 3 and 4 are satisfied, for $t_1 \in [\bar{T}, \infty)$,

(a) if $\theta \in [\hat{\theta}(\Delta), \bar{\theta}]$ where $\hat{\theta}(\Delta) = 1 - e^{(r+\rho)\Delta} \left[1 - (r+\rho) \frac{1-e^{-r\Delta}}{r} \frac{4A}{2A+x} \right]$, both firms invest at $\bar{T}$;

(b) if $\theta \in [1 - \frac{r+\rho}{r} e^{\rho\Delta} + \frac{\rho}{r} e^{(r+\rho)\Delta}, \hat{\theta}(\Delta))$, both firms invest at:

$$T^{le} = t_0 - \frac{1}{\rho} \ln \left(\frac{2A + x}{9b\gamma(r + \rho)} \right). \tag{14}$$

(c) if $\theta \in [0, 1 - \frac{r+\rho}{r} e^{\rho\Delta} + \frac{\rho}{r} e^{(r+\rho)\Delta})$, the subgame perfect equilibrium is either T^{le}, or it is such that it cannot be subgame perfect in the interval $t_1 \in [t_0, \infty)$.

Proof: See the Appendix.

When the spillover is low, the scope for waiting Δ before investing is limited and hence $\bar{T}$ is low. Therefore, for a low θ the payoff-maximizing choice for the adoption time is unconstrained and thus the late equilibrium is given by (14).

3.3 Equilibrium selection

As already remarked, subgame perfectness is the criterion we use to select among the equilibria identified in the previous Sub-section. Subgame perfectness requires that the equilibrium must survive all the possible off-equilibrium deviations. Accordingly, in the present context, the equilibrium selection must be carried out comparing the leader's payoff at any candidate equilibrium, with her payoff at any adoption date earlier than the one that is part of the equilibrium. Unfortunately, this task cannot be performed analytically, due to the high degree of non linearity in our model. Hence, we now present some numerical results.[20]

In our simulations, we normalize to unity the market dimension parameter A, and we fix the discount rate r to 0.03, which is consistent with computing calendar time in years. The parameter γ does not play any substantial role: the effect of an higher γ (i.e. of a less efficient R&D) is to postpone all of the equilibria, without changing their relative convenience. Likewise, the choice for b is inconsequential: an increase in b always induces a proportional contraction in per period profits. Hence, we choose $b = 1$ and $\gamma = 150$ with no loss of generality. As for ρ, we study−in the schumpeterian tradition−industry-specific rates of reduction in innovation costs. Industry I is technologically mature, but it still benefits from some technical progress in the sectors producing its machinery. Accordingly, $\rho = 0.01$. In

[20] Our routine has been written in Gauss, and it is based on a discretization of the space [θ x Δ], for $\theta \in [10^{(-10)}, 0.8]$ and $\Delta \in [10^{(-10)}, 3]$. We have used 240.000 gridpoints, however our results do not relevantly change for any number of evaluation points larger than 15.000. This routine is available upon request from the authors.

industry II, $\rho = 0.05$, which is the case of a fairly dynamic sector. Finally, industry III is a sector involved in a "technical revolution", where $\rho = 0.09$. To appreciate our figures, consider that the average economy-wide increase in productivity is of the order of 2% a year; moreover consider that the cost-effective technical progress parameter, ρ, may well be lower than the productivity growth rate, due to increases in the real wage in the research sectors.

To preserve the duopolistic structure of our market, we consider only non-drastic innovation (Assumption 1). Hence, the size of the R&D output, x, is lower than A ($x < 1$). We investigate two types of innovative output: a moderate innovation where $x = 0.05A(= 0.05)$ and a major innovation where $x = 0.5A(= 0.5)$.

Figure 4 portrays the equilibria arising in the case of a moderate innovation. Panel (a) highlights that in Industry I a low spillover implies, for a given Δ, a late equilibrium, while as the spillover increases the intermediate equilibrium prevails. For instance, when $\Delta = 2$, (refer to Table 1) the late equilibrium prevails when $\theta \leq 0.047$, while if $\theta > 0.047$ we have the intermediate equilibrium.

[Figure 4 about here]

The intuition for this result is the following: as underscored by Fudenberg and Tirole (1985), the smaller the cost reduction, the weaker is the incentive to innovate first.[21] Hence, a small x means that the highest deviation payoff for an early innovator is low, so that the early equilibrium never prevails over the late one. Moreover, a low spillover gives rise to a late equilibrium because it shrinks the intermediate region, since the second firm has a weak incentive to wait Δ to enjoy a modest R&D cost-reducing spillover (refer to the definition for $\bar{T}$ and to Figure 3, panel (c)). Hence, the late equilibrium prevails over any possible deviation occurring in the intermediate period.

When θ grows, the intermediate region enlarges, leading to a situation in which the first firm's deviation payoff becomes greater than her late equilibrium payoff. This leads to the prevalence of the intermediate equilibrium.

Panel (b) in Figure 4 shows the equilibria arising in Industry II. Again, for a given Δ, if the spillover is very low, the equilibrium in the R&D stage is the late one, for the same reasons as explained before. However, as θ increases (but it is still lower than $\theta''(\Delta)$), the early equilibrium prevails.

[21] This happens because the single innovator profit function, π_1^{10}, is more convex in x than π_1^{11} (see Sub-section 2.1).

Industry	Innovation			
	moderate		major	
I	$\theta \leq 0.047$	late	$\theta \leq 0.101$	early
	$\theta > 0.047$	intermediate	$\theta > 0.101$	intermediate
II	$\theta \leq 0.056$	late	$\theta \leq 0.104$	early
	$0.056 < \theta \leq 0.063$	early		
	$\theta > 0.063$	intermediate	$\theta > 0.104$	intermediate
III	$\theta \leq 0.056$	late	$\theta \leq 0.131$	early
	$0.056 < \theta \leq 0.079$	early		
	$\theta > 0.079$	intermediate	$\theta > 0.131$	intermediate

Table 1: R&D equilibria ($\Delta = 2$)

This happens in the small area contained between the two curves exiting from the origin in Figure 4 (refer also to Table 1, which is drawn for $\Delta = 2.$). To understand this result, bear in mind that an increase in ρ raises the payoffs in the intermediate region, because the R&D costs are lower.[22] The increase in the deviation payoff in the intermediate region destroys the late equilibrium, and moves the equilibrium to the early stage, because $\theta-$being lower than $\theta''(\Delta)-$is still small enough so that the intermediate equilibrium is dominated by the early one (refer to Sub-section 3.2.2).

Finally, a further increases in θ (above $\theta''(\Delta)$), drives us into the region in which the intermediate equilibrium exists; moreover, an increase in θ, reducing the first innovator's payoff in the early stage, makes the intermediate equilibrium dominant.

Figure 4, panel (c) shows the equilibrium selection in Industry III ($\rho = 0.09$): we have the same pattern observed for Industry II, with the only difference being that the θ threshold that discriminates the intermediate equilibrium from the early one is higher. This happens because the payoffs are higher in the early region than in the intermediate one. In fact, the former payoffs benefit more from a rapid technical progress.

The case of a major innovation is portrayed in Figure 5. There, $x = 0.50A(= 0.50)$. Here, the late equilibrium never prevails: a high x favors the selection of the early equilibrium. However, in our framework, an early equilibrium arises only for moderate values of the spillover parameter. In fact, when θ increases so that the intermediate equilibrium exists, the latter prevails for two reasons. First, a high θ negatively influences the first innovator payoffs in the early interval, because it anticipates the follower's investment date (equation (11)). Second, in the intermediate interval, as the spillover increases, the second comer's payoff gets larger, softening the incentive to

[22] The effect of ρ on the late equilibrium payoff is of course similar, but it is less significant since at that time the R&D cost are already very low.

invest for the leader. This milder competition implies higher payoffs for both firms, inducing the selection of the intermediate equilibrium.

[Figure 5 about here]

In sum, our analysis of the equilibrium selection process suggests that the intermediate equilibrium is the subgame perfect one in large portions of the parameter space.

This may help to explain the results in Schmidt-Dengler (2006). He estimates the determinants of the adoption of equipment for magnetic resolution images, which allows him to disentangle the preemption from the stand alone profit-maximizing effect. He finds that preemption accounts only for a relatively small share of the acceleration of investment timing that caracterizes the duopolistic market solution when compared to the collusive scenario. This is what our model prescribes for $\theta > \theta''(\Delta)$.

4 Welfare analysis

Having characterized the market equilibria, we can now analyze the benevolent planner problem.

In dealing with this issue, we introduce some hypotheses. First, we adopt a second best perspective, assuming that neither the number of firms acting in the market nor the way they compete in the second stage quantity game lies within the regulatory power of the benevolent planner. Hence, what this non-omnipotent planner can choose, is the timing of innovation.[23] Therefore, its decisions will be based on the instantaneous welfare levels – computed in Sub-section 2.1 – that have been obtained under the Cournot decentralized solution.

Second, the spillover obtained by firms engaging in a joint R&D project at the dates induced by the planner, is the same that is grasped by the second entrant when he waits Δ (i.e., the joint R&D activity grants a faster information flow, but not a cost advantage, when compared to a decentralized solution). The innovation costs incorporate the expenditures for the training of the employees required by the new production process, for some new machineries (or for adaptation of the existing plant), and so on (see De

[23] This approach is standard in the literature: see Stenbacka and Tombak (1994), Hoppe (2000), and Weeds (2002). The first best equilibrium for an omnipotent planner implies the presence of only one firm: whenever there are non-decreasing returns in the innovation size or probability, it is optimal to have only one firm to innovate and cover the entire market at the marginal (post-innovation) cost.

Bondt (1996)). Hence, the spillover parameter is not significantly increased by an R&D agreement.[24]

Therefore, the social planner maximizes – with respect to the adoption dates t_1 and t_2 – the following welfare function

$$\begin{aligned} W_1(t_1,t_2) &= \int_{t_0}^{t_1} W^{00} e^{-r(t-t_0)} dt + \int_{t_1}^{t_2} W^{10} e^{-r(t-t_0)} dt + \int_{t_2}^{\infty} W_1^{11} e^{-r(t-t_0)} dt \\ & -\gamma x e^{-(r+\rho)(t_1-t_0)} - (1-\theta)\gamma x e^{-(r+\rho)(t_2-t_0)}, \end{aligned} \tag{15}$$

where the (second best) instantaneous welfare levels, are given by Eqs. (2), (4) and (6), and $t_1 \leq t_2$ is a natural constraint.

The maximization of (15) yields

$$T_1^{SP} = t_0 - \frac{1}{\rho} \ln\left(\frac{8A+11x}{18b\gamma(r+\rho)}\right), \quad T_2^{SP} = t_0 - \frac{1}{\rho} \ln\left(\frac{8A-3x}{18b\gamma(r+\rho)(1-\theta)}\right),$$

if $\theta \leq \frac{14x}{8A+11x}$, and

$$T_1^{SP} = T_2^{SP} = T^{SP} = t_0 - \frac{1}{\rho} \ln\left(\frac{8A+4x}{9b\gamma(r+\rho)(2-\theta)}\right),$$

if not. The superscript SP stands for "social planner".

To verify whether the decentralized solution induces overinvestment in comparison with the centralized one, we compare the discounted (to t_0) innovation costs implied by the subgame perfect market solution with those obtained by the social planner. When the market innovation costs are higher (lower) than the planner solution ones, there is overinvestment (underivestment). The difference in the firms' timings that characterize the centralized solution and the decentralized one adds to the inefficiency related to the use of a non-optimal amount of resources.

Because the market game often does not have a closed form solution, to appreciate the differences in the discounted innovation costs, we need to rely on numerical simulations, which allow to obtain the following results:

i) Whenever the early equilibrium prevails, the market solution implies an excessive use of resources (i.e. overinvestment).

ii) Symmetrically, when the late equilibrium is subgame perfect, the decentralized solution involves a too low level of investment.

[24] The alternative assumption of an increasing θ will be briefly discussed in footnote 26.

iii) When the intermediate equilibrium dominates, it implies underinvestment, but for a small parameters sub-set when the size of the innovation is small, and the speed of the exogenous technical progress is high.

While the first two results are intuitive, the third deserves more attention.

To understand why an overinvesting intermediate equilibrium is possible only if the innovation size is small, consider the analysis in Sub-section 2.1. There, we have shown that both the instantaneous social welfare, and the firms profits increase more than proportionally with the size of the innovation. Because the social welfare is larger than the firms profit, also the wedge between the social and the private incentives to innovate increases with x, which acts against the possibility of overinvestment with a large innovation.

An increase in ρ reduces both the social planner's optimal adoption date(s) and the intermediate equilibrium ones. In the market game a steeper cost reduction profile, has strong effects on the innovation dates. In fact, given the leader's optimal timing, a faster cost reduction benefits the follower's payoff. This provides an incentive for his preemptive behavior, which may lead to overinvestment.

Accordingly, the portion of the parameter space with overinvestment is the widest, the largest is ρ, and the lowest is x. However, even in this case, the overinvestment area is very small: e. g. for $\rho = 0.09$, $x = 0.05$, and $\Delta = 2$, the intermediate equilibrium implies overivestment for $\theta \in [0.080, 0.115]$.[25]

Hence, not only the intermediate equilibrium prevails for most of the parameter configurations (as shown in Sub-section 3.3), but it also implies that the duopolistic market equilibrium involves underinvestment. This applies even when the innovation size is large, and hence the incentives to hasten innovation are remarkable. Therefore, the market equilibrium calls for public policies aimed at increasing the research activity even in this case, unless the inter-firm spillover is very low. Notice that the natural indicators of a highly competitive environment, namely a diffusion equilibrium and rent equalization, do not necessarily imply that the R&D investment is excessive from the social planner's perspective.

When we focus on minor innovations – the case in which the market equilibrium underinvests, according to the earlier literature – our result imply that the policies aimed at stimulating R&D have to be less sizeable than suggested before, because the underinvesting intermediate equilibrium

[25]Notice that Assumption 6 applies only if $\theta > \theta'''(\Delta)$, i.e. when the intermediate equilibrium already implies underinvestment. Hence, it is not crucial for our results.

is closer to the social optimum than the late equilibrium.[26]

5 Conclusions

In our duopoly game, firms, in addition to a technological externality, takes into account a spillover that lowers the second comer's innovation cost. This spillover exerts its effect after a "disclosure lag". In this setting, a new equilibrium arises, in which the R&D investment takes place at intermediate dates in comparison with those already identified in the literature.

Preemption, R&D diffusion, and the possibility of rent equalization characterize the intermediate equilibrium, which is competitive, although in a mild form. The intermediate equilibrium is subgame perfect for a large range of the parameters set; moreover, it is socially inefficient, implying a low level of investment in R&D.

This happens even in presence of major innovations, despite the large incentive to invest in R&D provided by this type of innovation. This result has important implications for innovation policy. For example, research joint ventures should be assessed in more favorable terms than those implied by the literature following d'Aspremont and Jacquemin (1988), and Kamien, Muller, and Zhang (1992). In fact, while a RJV may underinvest in comparison to an highly competitive equilibrium, it is likely to improve social welfare over a 'mildly competitive', underinvesting, market outcome. Furthermore, our paper suggests that R&D subsidies should be set in place in a range of market configurations wider than that has been previously proposed. Finally, our analysis provides an argument against the use of entry regulations (or price caps), which are sometimes used to slow technology adoption, e.g. in telecommunication industries. We leave the analysis of these policy instruments for further research.

When the innovation size is small, the prevalence of the intermediate equilibrium implies that R&D enhancing policies must be less intense than devised in the earlier literature. Actually, policies designed without taking into account the inter-firm spillover can be largely oversized, even when the spillover is quantitatively modest. Notice also that the intermediate equilibrium calls for moderate policies, which may prove easy to implement

[26] Suppose that a joint R&D activity guarantees not only a faster but also an easier, and hence less costly, information flow. In this case the spillover parameter in Eq. (15) should be higher than in the market game, and the social planner should dictate earlier investment date(s). Under this alternative assumption, result ii) is unaffected, result iii) is strenghtened, because it applies for even larger parameters set, while result i) weakens. In fact, it is possible – for a sizeable (and somehow irrealistic) increase in θ – that the second best optimal timing anticipates the early equilibrium ones.

from a political economy perspective.

Our setting can be extended in various directions, which however, would require an heavy use of numerical techniques. For example, it would be interesting to consider a stochastic inter-firm spillover, in which the probability of information diffusion depends upon the time elapsed from the introduction of the innovation, and on the follower's imitation effort. Also, we would like to consider the possibility that the leader actively (and hence costly) attempts to prevent information leakages, thereby increasing the disclosure lag. Whenever the firms' efforts lenghten this lag, they reduce the follower's equilibrium payoff, and hence, also the leader's one. Therefore, they tend to reduce the intermediate equilibrium dominance area. However, the analysis developed in Section 3.3 suggests that the effect of the disclosure lag on the dominance areas are weak. Hence, our main result should not be undermined by the adoption of a richer framework.

References

Beath, J., Katsoulacos, Y. and Ulph, D. "Strategic R&D Policy", *Economic Journal*, Vol. 99 (1989), pp. 74-83.

Cohen, W.M., Goto A., Nagata A., Nelson R.R., and Walsh, J.P., "R&D spillovers, patents and the incentives to innovate in Japan and the United States", *Research Policy*, Vol. 31 (2002), pp. 1349-1367.

Dasgupta, P., "Patents, priority and imitation or, the economics of races and waiting games." *Economic Journal*, Vol. 98 (1988), pp. 66–80.

d'Aspremont, C., and Jacquemin, A., "Cooperative and Non-cooperative R&D in Duopoly with Spillovers." *American Economic Review*, Vol. 78 (1988), pp. 1133–1137.

De Bondt, R., "Spillovers and innovative activities." *International Journal of Industrial Organization*, Vol. 15 (1996), pp. 1–28.

Delbono, F. and Denicolò, V. "Incentives to innovate in a Cournot oligopoly." *Quarterly Journal of Economics*, Vol. 105 (1991), pp.950-961.

Denicolò, V., "Patent Races and Optimal Patent Breadth and Length." *Journal of Industrial Economics*, Vol. 44 (1996), pp. 249–265.

Dutta, P. K., Lach, S., and Rustichini, A., "Better Late Than Early: Vertical Differentiation in the Adoption of a New Technology " *Journal of Economics and Management Strategy*, Vol. 4 (1995), pp. 563-89.

Fudenberg, D., and Tirole, J., "Preemption and Rent Equalization in the Adoption of New Technology." *Review of Economic Studies*, Vol. 52 (1985), pp. 383–401.

— *Game Theory*. Cambridge, MA: MIT Press, 1991.

Hernan, R., Marin, P.L., and Siotis, G., "An Empirical Evaluation of the Determinants of Research Joint Venture Formation." *Journal of Industrial Economics*, Vol. 51 (2003), pp. 75–89.

Hoppe, H.C., "Second-mover advantages in the strategic adoption of new technology under uncertainty." *International Journal of Industrial Organization*, Vol. 18 (2000), pp. 315–338.

Hoppe, H.C., "The timing of new technology adoption: theoretical models and empirical evidence." *The Manchester School*, Vol. 70 (2002), pp. 56-76.

Hoppe, H.C., Lehmann-Grube, U., "Innovation Timing Games: A General Framework with Applications." *Journal of Economic Theory*, Vol. 121 (2005), pp. 30–50.

Jin, J.Y., and Troege, M., "R&D Competition and Endogenous Spillovers." *The Manchester School*, Vol. 74 (2006), pp. 40–51.

Kamien, M.I., Muller, E., and Zhang, I., "Research Joint Ventures and R&D Cartels." *American Economic Review*, Vol. 82 (1992), pp. 1293–1306.

Katz, M.L., and Shapiro, C., "R&D Rivalry with Licensing or Imitation." *American Economic Review*, Vol. 77 (1987), pp. 402–420.

Lee, T. and Wilde, L., "Market structure and innovation: a reformulation." *Quarterly Journal of Economics*, Vol. 94 (1980), pp.429-36.

Loury, G., "Market structure and innovation." *Quarterly Journal of Economics*, Vol. 93 (1979), pp.395-410.

Mansfield, E., "How Rapidly Does New Industrial Technology Leak Out?" *Journal of Industrial Economics*, Vol. 34 (1985), pp. 217–223.

Mansfield, E., Schwartz, M., and Wagner, S., "Imitation Costs and Patents: An Empirical Study." *Economic Journal*, Vol. 91 (1981), pp. 907–918.

Reinganum, J., "On the diffusion of new technologies: a game theoretic approach." *Review of Economic Studies*, Vol. 48, pp. 395-405.

Riordan, M.H., "Regulation and preemptive technology adoption." *RAND Journal of Economics* Vol. 23 (1992), pp.334-349.

Schmidt-Dengler, P., "The Timing of New Technology Adoption: The Case of MRI", *mimeo LSE*, (2006).

Stenbacka, R. and Tombak, M.H., "Strategic timing of adoption of new technologies under uncertainty." *International Journal of Industrial Organization*, Vol. 12 (1994), pp. 387–411.

Weeds, H., "Strategic Delay in a Real Options Model of R&D Competition." *Review of Economic Studies*, Vol. 69 (2002), pp. 729–747.

APPENDIX

Proof of Proposition 1

As a preliminary, notice that Assumption 4 guarantees that the interval $[t_0, T_2^* - \Delta]$ is non empty for $\theta \in [0, \bar{\theta}]$. Notice, moreover, that Assumption 2 implies that all the four sub-cases in Proposition 1 are well defined. This is because Assumption 2 can be written as: $\bar{\theta} \geq \max\left\{\frac{x}{A+x}, \theta''(\bar{\Delta})\right\}$.

Proof of part (a). As it is standard, we start characterizing the optimal strategy for the follower. When the first firm has sunk the innovation cost at time $t_1 \in [t_0, T_2^* - \Delta]$, the payoff at time t_0 for the second firm, when it invests at t_2, is given by (10).

Suppose that the second comer decides to wait more than Δ, to grasp the inter-firms spillover; in this case Eq. (8) prescribes that the innovation cost is $C_2(T_2) = (1-\theta)\gamma x e^{-\rho(T_2 - t_0)}$, and a few straightforward calculations show that T_2^*, as given by (11), maximizes $V_2(t_1, t_2)$.

Alternatively, the second comer could decide not to wait for Δ periods, and in this case he should invest at:

$$T_2' = t_0 - \frac{1}{\rho}\ln\left(\frac{4A}{9b\gamma(r+\rho)}\right). \tag{A.1}$$

This second alternative requires that $T_2' \in [t_1, t_1 + \Delta)$. Had the latter restriction not been satisfied, the innovation follower would have benefited from the spillover. Since $T_2' > T_2^*$, whenever $t_1 \in [t_0, T_2^* - \Delta]$ the innovation follower grasps the imitation benefits and invests at T_2^*. Because of this, his payoff can be written as:

$$\begin{aligned} V_2(t_1, T_2^*) &= \frac{A^2}{9br} - \left[\frac{(2A-x)x}{9br}\right] e^{-r(t_1 - t_0)} \\ &\quad + \frac{\rho}{r(r+\rho)}\frac{4Ax}{9b}\left(\frac{4A}{9b\gamma(r+\rho)(1-\theta)}\right)^{\frac{r}{\rho}}, \end{aligned} \tag{A.2}$$

which implies: $\frac{\partial V_2(t_1, T_2^*)}{\partial t_1} > 0$, and $\frac{\partial^2 V_2(t_1, T_2^*)}{(\partial t_1)^2} < 0$ in the whole interval $[t_0, T_2^* - \Delta]$. Also notice that $\frac{\partial V_2(t_1, T_2^*)}{\partial \theta} > 0$ for every $t_1 \in [t_0, T_2^* - \Delta]$.

Having determined the optimal decision for the follower, we now determine the leader's best strategy. When $t_1 \in [t_0, T_2^* - \Delta]$, the innovation leader payoff is given by (9) in which the innovation costs are provided by (7) and $t_2 = T_2^*$. Exploiting equation (11), we obtain:

$$V_1(t_1, T_2^*) = \frac{A^2}{9br} + \left[\frac{4(A+x)x}{9br} - \gamma x e^{-\rho(t_1-t_0)}\right] e^{-r(t_1-t_0)}$$
$$-\frac{(2A+3x)x}{9br}\left(\frac{4A}{9b\gamma(r+\rho)(1-\theta)}\right)^{\frac{r}{\rho}}. \qquad \text{(A.3)}$$

Hence, $\frac{\partial V_1(t_1,T_2^*)}{\partial t_1} \gtreqless 0$ when $t_1 \lesseqgtr T_1^*$ (with T_1^* given by (12)). Notice that T_1^*, in general, need not be smaller than $T_2^* - \Delta$. Notice also that $\frac{\partial V_1(t_1,T_2^*)}{\partial \theta} < 0$ for every $t_1 \in [t_0, T_2^* - \Delta]$.

When $\theta \leq \theta'(\Delta)$, it is easy to show that $T_1^* \leq T_2^* - \Delta$, because the latter inequality requires $A + x \geq \frac{A}{1-\theta} e^{\rho\Delta}$, and hence $\theta \leq 1 - \frac{A}{A+x} e^{\rho\Delta}$. We now check whether – when $t_1 = T_1^*$– the leader's payoff is larger than the follower's one. Exploiting equation (A.3), $V_1(T_1^*, T_2^*)$ can be easily written as:

$$V_1(T_1^*, T_2^*) =$$
$$= \frac{A^2}{9br} + \frac{4\rho(A+x)x}{9br(r+\rho)}\left(\frac{4(A+x)}{9b\gamma(r+\rho)}\right)^{\frac{r}{\rho}} - \frac{(2A+3x)x}{9br}\left(\frac{4A}{9b\gamma(r+\rho)(1-\theta)}\right)^{\frac{r}{\rho}},$$

while, from (A.2), $V_2(T_1^*, T_2^*)$ is:

$$V_2(T_1^*, T_2^*) =$$
$$= \frac{A^2}{9br} - \frac{(2A-x)x}{9br}\left(\frac{4(A+x)}{9b\gamma(r+\rho)}\right)^{\frac{r}{\rho}} + \frac{\rho}{r(r+\rho)}\frac{4Ax}{9b}\left(\frac{4A}{9b\gamma(r+\rho)(1-\theta)}\right)^{\frac{r}{\rho}}.$$

A few calculations allow us to show that $V_1(T_1^*, T_2^*) \geq V_2(T_1^*, T_2^*)$ if $\theta \leq \tilde{\theta}$. Notice moreover that $\tilde{\theta} \leq \theta'(\Delta)$ for $\Delta \in [0, \Delta']$, where

$$\Delta' = \frac{1}{r}\ln\left[1 + \frac{r4x}{\rho 3(2A+x) + r(2A-x)}\right] > 0.$$

Hence, $V_1(T_1^*, T_2^*) \geq V_2(T_1^*, T_2^*)$ and $T_1^* < T_2^* - \Delta$ if $\theta \leq \min\{\tilde{\theta},\, \theta'(\Delta)\}$. Because $\Delta' > 0$, and $\theta'(\Delta) > 0$, we have that $\tilde{\theta} > 0$, which guarantees that the region $\theta \leq \min\{\tilde{\theta},\, \theta'(\Delta)\}$ is non-empty.

To conclude that the first firm equilibrium adoption date is $\max\{\underline{T}_1, t_0\}$, we follow the argument developed in Fudenberg and Tirole (1985): when $V_1(T_1^*, T_2^*) > V_2(T_1^*, T_2^*)$, it is in each firm's interest to adopt at time T_1^* if the other firm has not adopted up to that time. But if a firm knows that the other will adopt at time T_1^*, it is in its interest to preempt at time $T_1^* - dt$, whenever $V_1(T_1^* - dt, T_2^*) \geq V_2(T_1^*, T_2^*)$. By backward induction, we conclude that the equilibrium strategy for the first innovator is $\max\{\underline{T}_1, t_0\}$, where $\underline{T}_1$ is the earliest adoption date such that $V_1(\underline{T}_1, T_2^*) \geq V_2(\underline{T}_1, T_2^*)$. In this case no firm wants to invest and anticipate the other to avoid be preempted later on. Mixed strategy equilibria are ruled out by Assumption 4. Hence, Part (a) is proved.

We now consider case (b), i.e. $\theta'(\Delta) < \theta \leq \theta''(\Delta)$. A few calculations show that the restriction $\theta \leq \theta''(\Delta)$, implies: $V_1(T_2^* - \Delta, T_2^*) \geq V_2(T_2^* - \Delta, T_2^*)$. Hence, the preemption argument sketched above applies again and the equilibrium is $\{\max\{\underline{T}_1, t_0\}, T_2^*\}$ (refer to Figure 1, panel (b)). Notice that, at $\Delta = \Delta'$, $\theta'(\Delta') = \theta''(\Delta')(= \tilde{\theta})$. Therefore, this case applies only when $\Delta \geq \Delta'$. This completes the proof of Part (b).

To prove Part (c), split the restriction $\theta > \max\{\theta'(\Delta), \theta''(\Delta)\}$, into $\theta > \theta'(\Delta)$, and $\theta > \theta''(\Delta)$ (which must hold simultaneously). The restriction $\theta > \theta'(\Delta)$ implies that $T_1^* > T_2^* - \Delta$ and hence that $V_1(t_1, T_2^*)$ is increasing in the whole interval $t_1 \in [t_0, T_2^* - \Delta]$. In its turn, $\theta > \theta''(\Delta)$ implies $V_2(T_2^* - \Delta, T_2^*) > V_1(T_2^* - \Delta, T_2^*)$. Hence, it is in each firm's interest to wait until $T_2^* - \Delta$, while the preemption argument does not apply. By Assumption 6, this completes the proof of Part (c).

Finally, we analyze case (d), i.e. $\tilde{\theta} < \theta \leq \theta'(\Delta)$. The restriction $\theta \leq \theta'(\Delta)$ implies $T_1^* \leq T_2^* - \Delta$. However, if $\theta > \tilde{\theta}$, as shown in Part (a), $V_1(T_1^*, T_2^*) < V_2(T_1^*, T_2^*)$. Therefore, by Assumption 6, the first firm becomes the leader and invest at T_1^*, while the second invests at T_2^*. This completes the proof of Part (d).

Proof of Proposition 2

As a preliminary, notice that Assumption 2 guarantees that all the sub-cases in Proposition 2 are well defined.

Notice, moreover, that $\bar{T} > T_2^* - \Delta$ for any $\theta \in [0, \bar{\theta}]$.

Notice, finally, that some tedious calculations grant that: $\theta'''(\Delta) \geq \theta''(\Delta)$.

As before, we start characterizing the optimal strategy for the follower.

When $t_1 \geq T_2^* - \Delta$, the innovation follower will never wait more than Δ, simply because $t_1 \geq T_2^* - \Delta$. Hence, his available strategies are:

(1) wait exactly Δ periods to grasp the benefit of the spillover,

(2) invest immediately after the innovation leader, and

(3) wait for a time span shorter than Δ (to exploit the exogenous technological externality), and then invest (therefore, without exploiting the inter-firm spillover).

First we compare what the innovation follower obtains by waiting Δ periods (strategy 1) with what he gets by investing immediately after the innovation leader (strategy 2). Hence, we determine when $V_2(t_1, t_1 + \Delta) \geq V_2(t_1, t_1)$.

This inequality immediately boils down to:

$$\begin{aligned} & \frac{4Ax}{9br}e^{-r(t_1+\Delta-t_0)} - (1-\theta)\gamma x e^{-(r+\rho)(t_1+\Delta-t_0)} \\ \geq \;& \frac{4Ax}{9br}e^{-r(t_1-t_0)} - \gamma x e^{-(r+\rho)(t_1-t_0)}, \end{aligned}$$

which, in its turn, is satisfied when: $t_1 \leq \bar{T}$. Hence, the innovation follower never chooses to immediately follow the leader for any $t_1 \in [T_2^* - \Delta, \bar{T}]$.

Next, we compare strategy 1 with strategy 3.

In doing so, recall the definition of T_2' from (A.1), and distinguish the case $\bar{T} \geq T_2'$ from the case $\bar{T} < T_2'$. Notice that the inequality $\bar{T} \geq T_2'$ is satisfied when $\theta \geq 1 - \frac{r+\rho}{r}e^{\rho\Delta} + \frac{\rho}{r}e^{(r+\rho)\Delta}$, and some calculations allow us to verify that: $\theta''(\Delta) \geq 1 - \frac{r+\rho}{r}e^{\rho\Delta} + \frac{\rho}{r}e^{(r+\rho)\Delta}$.

Hence, in cases (a) and (b), $\bar{T} \geq T_2'$.

Suppose now that the leader invests at $t_1 \in [T_2^* - \Delta, T_2' - \Delta]$. In this interval, the payoff function for a follower who does not exploit the inter-firm spillover is always increasing. In fact, this function is concave with a global maximum at $t_2 = T_2'$ $\forall$ t_1(refer to the Proof for Proposition 1). Hence, it is optimal for the follower to invest later than $T_2' - \Delta$, which implies that the spillover is actually exploited.

When $t_1 \in (T_2' - \Delta, T_2']$, the optimal strategy for the innovation follower must be determined by comparing what it gets by delaying its investment for Δ periods with what can be obtained by investing at T_2'. Hence, we need to determine when $V_2(t_1, t_1+\Delta) - V_2(t_1, T_2') \geq 0$. This inequality immediately boils down to:

$$\begin{aligned} & \frac{4Ax}{9br}\left[e^{-r(t_1+\Delta-t_0)} - e^{-r(T_2'-t_0)}\right] \\ & -\gamma x\left[(1-\theta)e^{-(r+\rho)(t_1+\Delta-t_0)} - e^{-(r+\rho)(T_2'-t_0)}\right] \geq 0. \end{aligned} \tag{A.4}$$

It is easy to show that the left hand side of (A.4) is non-increasing in t_1 in the whole interval $(T_2' - \Delta, T_2']$. Evaluate equation (A.4) at $t_1 = T_2'$, and–exploiting equation (A.1)–substitute out T_2' when convenient, to obtain:

$$e^{-r(T_2'-t_0)}\frac{4Ax}{9br}\left[e^{-r\Delta} - 1 - r(1-\theta)\frac{e^{-(r+\rho)\Delta}}{r+\rho} + \frac{r}{r+\rho}\right] \geq 0,$$

which is fulfilled when $\theta \geq 1 - \frac{r+\rho}{r}e^{\rho\Delta} + \frac{\rho}{r}e^{(r+\rho)\Delta}$. Hence, under this restriction, the follower's strategy of waiting Δ periods is chosen for any $t_1 \in (T_2' - \Delta, T_2']$.

Finally, strategy 3 can never be optimal for $t_1 \in (T_2', \bar{T}]$ simply because the payoff function for a follower who does not exploit the spillover is decreasing in $t_2 \in (t_1, \bar{T}]$ and thus there is no point in waiting when the leader has already invested; recall moreover that the immediate investment strategy has already been proven to be dominated by a time Δ delay.

Hence, in cases (a) and (b) the follower's optimal reply to the innovation leader's decision to invest is to wait exactly Δ periods to grasp the inter-firm spillover and then invest.

The analysis for case (c) must be splitted into two sub-cases.

c1) When $\theta \in [1 - \frac{r+\rho}{r} e^{\rho\Delta} + \frac{\rho}{r} e^{(r+\rho)\Delta}, \theta''(\Delta))$, then $\bar{T} \geq T_2'$ and the analysis developed above applies.

c2) When $\theta \in [0, 1 - \frac{r+\rho}{r} e^{\rho\Delta} + \frac{\rho}{r} e^{(r+\rho)\Delta})$, then $\bar{T} < T_2'$. Notice, however, that it is possible to prove that $T_2' - \Delta < \bar{T}$. In the time interval $t_1 \in [T_2^* - \Delta, T_2' - \Delta]$ the optimal strategy is again to wait Δ and exploit the inter-firm spillover, because the follower's payoff function $V_2(t_1, t_2)$ is increasing in $t_2 \in [t_1, T_2' - \Delta]$.

When $t_1 \in (T_2' - \Delta, \bar{T}]$, the optimal strategy for the innovation follower must be determined by comparing what he gets by delaying his investment for Δ periods with what can be obtained by investing at T_2'. Unfortunately, it is not possible to characterize analytically the sub-intervals in which the two alternative strategies prevail. Let us denote by $\breve{T}_1$ the instant when $V_2(t_1, t_1 + \Delta) = V_2(t_1, T_2')$. $\breve{T}_1 \in (T_2' - \Delta, \bar{T}]$ because: $V_2(t_1, t_1 + \Delta) - V_2(t_1, T_2')$ is non-increasing in t_1; $\lim_{\epsilon \to 0} [V_2(T_2' - \Delta + \epsilon, T_2' + \epsilon) - V_2(T_2' - \Delta + \epsilon, T_2')] > 0$ and $V_2(\bar{T}, \bar{T} + \Delta) - V_2(\bar{T}, T_2') < 0$, in fact, by definition $V_2(\bar{T}, \bar{T} + \Delta) = V_2(\bar{T}, \bar{T})$ and $V_2(\bar{T}, \bar{T}) < V_2(\bar{T}, T_2')$. Hence, for $t_1 \in (T_2' - \Delta, \breve{T}_1]$ strategy (1) is optimal, while for $t_1 \in (\breve{T}_1, T_2']$ the innovation follower decides to innovate at T_2' (strategy 3).

We conclude our characterization for the follower's optimal strategy by noting that $V_2(t_1, t_1 + \Delta)$ is a concave function with its maximum at the right of $T_2^* - \Delta$.

We now proceed to analyzing the first firm's behavior.

If $\theta \in (\theta'''(\Delta), \bar{\theta}]$ (case a), the first innovator is aware of the fact that her competitor will always invest with a delay of Δ periods. Hence, she computes her payoff for $t_1 \in [T_2^* - \Delta, \bar{T}]$ which is:

$$V_1(t_1, t_1 + \Delta) = \frac{A^2}{9br} +$$
$$\left\{ \left[\frac{4(A + x)x}{9br} - \gamma x e^{-\rho(t_1 - t_0)} \right] - \frac{(2A + 3x)x}{9br} e^{-r\Delta} \right\} e^{-r(t_1 - t_0)}.$$

Notice that $\theta \geq \theta''(\Delta)$ implies that $V_2(T_2^* - \Delta, T_2^*) \geq V_1(T_2^* - \Delta, T_2^*)$, hence the latter inequality applies also for $\theta \geq \theta'''(\Delta)$; notice moreover that the payoff function $V_1(t_1, t_1 + \Delta)$ reaches its maximum at $t_1 = \hat{T}_1$.

Consider now the equation: $V_1(t_1, t_1 + \Delta) = V_2(t_1, t_1 + \Delta)$, which has a unique solution: $T_1^{ip} = t_0 - \frac{1}{\rho} \ln \left(\frac{3(2A+x)(1-e^{-r\Delta})}{9br\gamma[1-(1-\theta)e^{-(r+\rho)\Delta}]} \right)$. Notice that $T_1^{ip} > \hat{T}_1$ if $\theta > \theta'''(\Delta)$, hence for $\theta \in (\theta'''(\Delta), \bar{\theta}]$, $T_1^{ip} \in [\hat{T}_1, \bar{T}]$. (This happens because the inter-firm spillover is very high, implying a high payoff for the second firm). Because the $V_2(t_1, t_1 + \Delta)$ curve lies above the $V_1(t_1, t_1 + \Delta)$ curve for $t_1 \in [T_2^* - \Delta, \hat{T}_1]$, no firm has an incentive to preempt its rival before $\hat{T}_1$. The first firm, under Assumption 6, has no incentive to delay her adoption beyond $\hat{T}_1$, because $V_1(t_1, t_1 + \Delta)$ is decreasing for $t_1 \in [\hat{T}_1, \bar{T}]$. Hence, in the interval $[T_2^* - \Delta, \bar{T}]$, the unique subgame perfect equilibrium is $t_1 = \hat{T}_1$, $t_2 = \hat{T}_1 + \Delta$. The first panel in Figure 3 portrays the situation analyzed here. This proves Part (a).

In case (b) ($\theta \in [\theta''(\Delta), \theta'''(\Delta)]$), the first innovator is aware, again, of the fact that her competitor will invest with a delay of Δ periods. In this case, however, $T_1^{ip} \in [T_2^* - \Delta, \hat{T}_1]$: in fact, with respect to the previous case, the reduction in the spillover parameter shifts downward the follower's payoff function (as depicted in Figure 3, panel (b)) and, for $t_1 \in [T_1^{ip}, \bar{T}]$ the $V_2(t_1, t_1 + \Delta)$ curve lies below the $V_1(t_1, t_1 + \Delta)$ curve. Hence, $t_1 = T_1^{ip}$ is part of the unique pure strategy equilibrium, due to the usual preemption argument. This proves Part (b).

When considering the follower's optimal strategy in case (c), we must distinguish again the two sub-cases: c1) $\theta \in [1 - \frac{r+\rho}{r} e^{\rho\Delta} + \frac{\rho}{r} e^{(r+\rho)\Delta}, \theta''(\Delta))$, and c2) $\theta \in [0, 1 - \frac{r+\rho}{r} e^{\rho\Delta} + \frac{\rho}{r} e^{(r+\rho)\Delta})$.

c1) When $\theta \in [1 - \frac{r+\rho}{r} e^{\rho\Delta} + \frac{\rho}{r} e^{(r+\rho)\Delta},\ \theta''(\Delta))$, the first innovator is aware of the fact that her competitor will invest with a delay of Δ periods. In this case one can show that the unique solution for the equation $V_1(t_1, t_1 + \Delta) = V_2(t_1, t_1 + \Delta)$, lies outside the interval $[T_2^* - \Delta, \bar{T}]$. (i.e. $T_1^{ip} < T_2^* - \Delta$). Because the follower's payoff is lower than the first innovator's one, in the whole interval $[T_2^* - \Delta, \bar{T}]$, and standard arguments imply the existence of a mixed strategy equilibrium, in which firms start to randomize at $T_2^* - \Delta$. This case is portrayed in panel (c) of Figure 3.

c2) When $\theta \in [0, 1 - \frac{r+\rho}{r} e^{\rho\Delta} + \frac{\rho}{r} e^{(r+\rho)\Delta})$, the first firm is aware that, if she chooses $t_1 \in [T_2^* - \Delta, \breve{T}_1]$, the follower picks an innovation time characterized by a delay of span Δ, while if she chooses $t_1 \in [\breve{T}_1, \bar{T}]$ the follower innovates at time $T_2' > \bar{T}$.

Suppose first that the follower innovates with a delay of Δ. In this case, because $T_1^{ip} < T_2^* - \Delta$, it is obvious that $V_1(t_1, t_1 + \Delta) > V_2(t_1, t_1 + \Delta)$ for any $t_1 \in [T_2^* - \Delta, \bar{T}]$ and hence, a fortiori, for any $t_1 \in [T_2^* - \Delta, \breve{T}_1]$. Suppose now that the follower innovates at time T_2'. In this case, again, $V_1(t_1, T_2') > V_2(t_1, T_2')$ for $t_1 \in [T_2' - \Delta, \bar{T}]$ and hence, a fortiori, for any $t_1 \in [\breve{T}_1, \bar{T}]$. (To show this, consider that $V_1(T_2', T_2') = V_2(T_2', T_2')$ and that $\partial[V_1(t_1, T_2') - V_2(t_1, T_2')]/\partial t_1 < 0$.). Hence, in the whole interval $[T_2^* - \Delta, \bar{T}]$, the follower's payoff is lower than the first innovator's one; again standard arguments imply the existence of a mixed strategy equilibrium, in which firms start to randomize at $T_2^* - \Delta$. This proves Part (c).

Proof of Proposition 3.

Notice, as a preliminary, that Assumption 2 guarantees that all the sub-cases in Proposition 3 are non-empty.

As usual, we start characterizing the optimal strategy for the follower.

The proof of Proposition 2 implies that the innovation follower will never wait Δ, for any $t_1 \geq \bar{T}$. Hence, his available strategies are:

(1) invest immediately after the innovation leader, and

(2) wait for a time span shorter than Δ (to exploit the exogenous technological externality), and then invest without exploiting the inter-firm spillover.

The proof of Proposition 1 implies that – when the innovation follower decides to wait – he invests at T_2', for any $t_1 \in (T_2' - \Delta, T_2']$. In fact, the payoff function for the follower, $V_2(t_1, t_2)$ has a maximum at T_2'.

In the proof for Proposition 2, we showed that, for $\theta \in [1 - \frac{r+\rho}{r} e^{\rho\Delta} + \frac{\rho}{r} e^{(r+\rho)\Delta}, \bar{\theta}]$, then $\bar{T} \geq T_2'$. Hence, under this parameter restriction, the second innovator invests immediately after the innovation leader. In fact, it is never in the follower's interest to wait Δ periods, because $t_1 > \bar{T}$, while the follower's payoff function is decreasing in t_2 in the whole interval $t_1 \in [\bar{T}, \infty)$. Hence, the follower has no point in waiting.

In contrast, when $\theta \in [0, 1 - \frac{r+\rho}{r} e^{\rho\Delta} + \frac{\rho}{r} e^{(r+\rho)\Delta})$, then $T_2' - \Delta < \bar{T} < T_2'$ (as shown in the proof for Proposition 2). Therefore, under this parameter restriction, the second innovator invests at T_2' when $t_1 \in [\bar{T}, T_2']$; when $t_1 \in (T_2', \infty)$, the innovation follower invests immediately after the innovation leader because its payoff function is decreasing in t_2.

We now analyze the first firm's behavior.

Suppose first, that $\theta \in [1 - \frac{r+\rho}{r} e^{\rho\Delta} + \frac{\rho}{r} e^{(r+\rho)\Delta}, \bar{\theta}]$, so that the first firm knows that – as soon as she innovates – the rival firm immediately sinks the innovation cost.

Hence, the payoff for the first firm is:

$$V_1(t_1,t_1) = \int_{t_0}^{t_1} \pi_1^{00} e^{-r(t-t_0)} dt + \\ + \int_{t_1}^{\infty} \pi_1^{11} e^{-r(t-t_0)} dt - \gamma x e^{-(r+\rho)(t_1-t_0)}. \qquad \text{(A.5)}$$

Maximization of (A.5) with respect to t_1 under the constraint $t_1 \geq \bar{T}$ yields that the first firm optimal timing is:

$\bar{T}$ if $\theta \geq \hat{\theta}(\Delta)$,

$T^{le} = t_0 - \frac{1}{\rho} \ln\left(\frac{2A+x}{9b\gamma(r+\rho)}\right)$ if $\theta < \hat{\theta}(\Delta)$.

It is now easy to show that $\hat{\theta}(\Delta) \geq 1 - \frac{r+\rho}{r} e^{\rho\Delta} + \frac{\rho}{r} e^{(r+\rho)\Delta}$. Accordingly, when $\theta \geq \hat{\theta}(\Delta)$, both firms invest at $\bar{T}$. This proves part (a).

To prove part (b), notice first that – when $\theta \in [1 - \frac{r+\rho}{r} e^{\rho\Delta} + \frac{\rho}{r} e^{(r+\rho)\Delta}, \hat{\theta}(\Delta))$– the second firm invests immediately after the first firm. Hence, the equilibrium is T^{le}.

Finally, consider part (c), which requires. $\theta \in [0, 1 - \frac{r+\rho}{r} e^{\rho\Delta} + \frac{\rho}{r} e^{(r+\rho)\Delta})$. When $t_1 \in [\bar{T}, T_2']$ the second firm invests at T_2'; when $t_1 \in [T_2', \infty)$, the second firm invests immediately after the first one. For the first firm, the optimal reply to T_2' would be to invest at $\bar{T}$. In fact, $\frac{\partial V_1(t_1,T_2')}{\partial t_1} < 0$ for $t_1 \in [\bar{T}, T_2']$ (differentiate $V_1(t_1, T_2')$ and check that $\bar{T} \geq T_1^*$). Notice that, for $t_1 \in [\bar{T}, T_2')$ the innovation leader's payoff is higher than the follower's one. (This is shown by noting that $V_1(T_2', T_2') = V_2(T_2', T_2')$, and that $\frac{\partial V_1(t_1,T_2')}{\partial t_1} \leq \frac{\partial V_2(t_1,T_2')}{\partial t_1}$ for $t_1 \in [T_2' - \Delta, T_2']$, and therefore, *a fortiori*, for $t_1 \in [\bar{T}, T_2']$.) The first mover advantage interval $t_1 \in [\bar{T}, T_2']$ gives rise to preemptive behaviors if $V_1(\bar{T}, T_2') \geq V_1(T^{le}, T^{le})$. Hence, in the interval $t_1 \in [\bar{T}, \infty)$, the equilibrium is T^{le} when $V_1(\bar{T}, T_2') < V_1(T^{le}, T^{le})$. Instead, when $V_1(\bar{T}, T_2') \geq V_1(T^{le}, T^{le})$, we have a mixed strategy equilibrium, in which firms randomize at $\bar{T}$. (However, notice that the preemption argument implies that such an equilibrium is not subgame perfect for $t_1 \in [t_0, \infty)$; in fact firms adopt earlier than at $\{\bar{T}, T_2'\}$. Refer to Figure 3, Panel (c)). This proves Part (c).

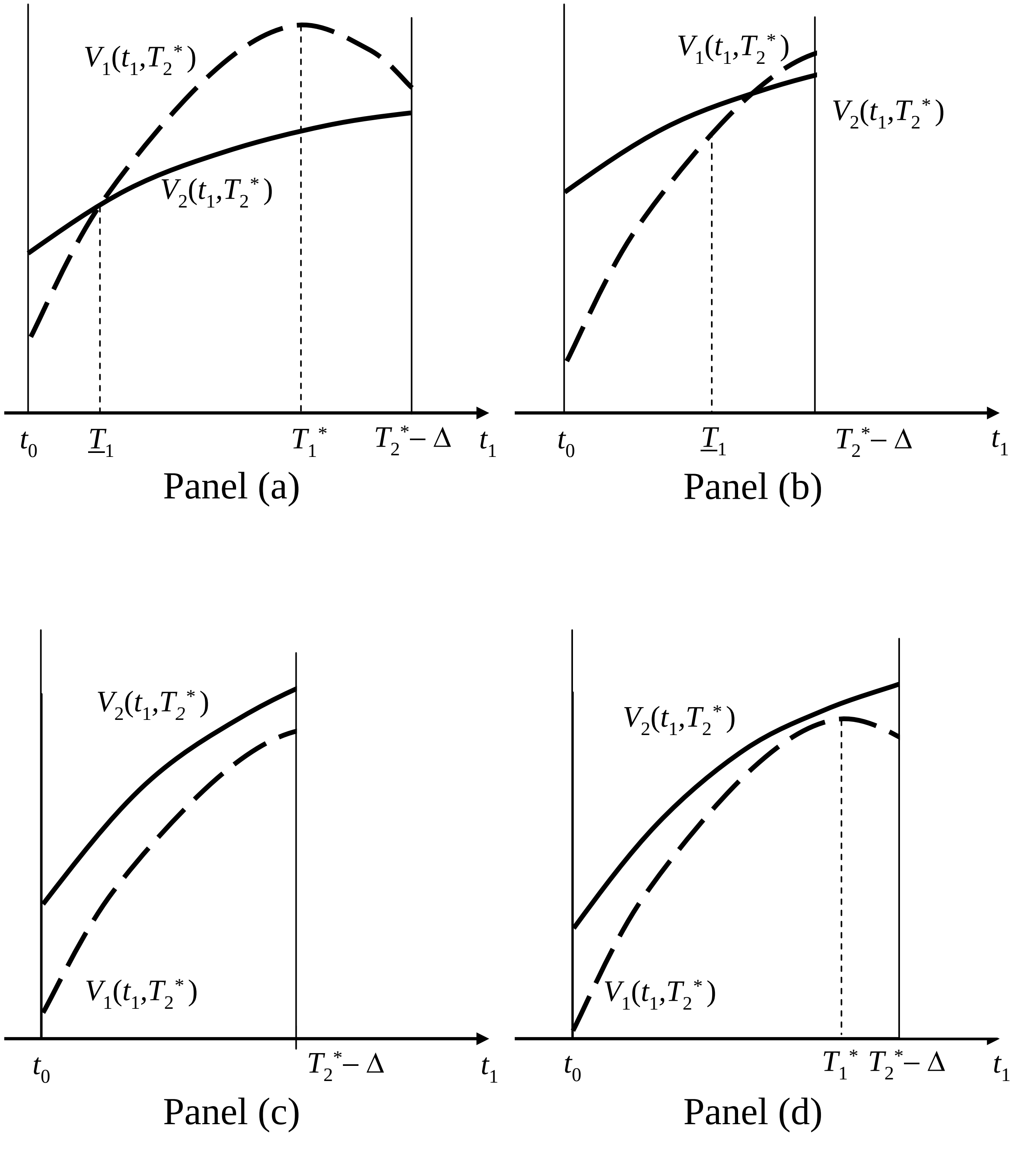


Figure 1: Alternative behaviors for the firms' discounted payoffs in the early interval

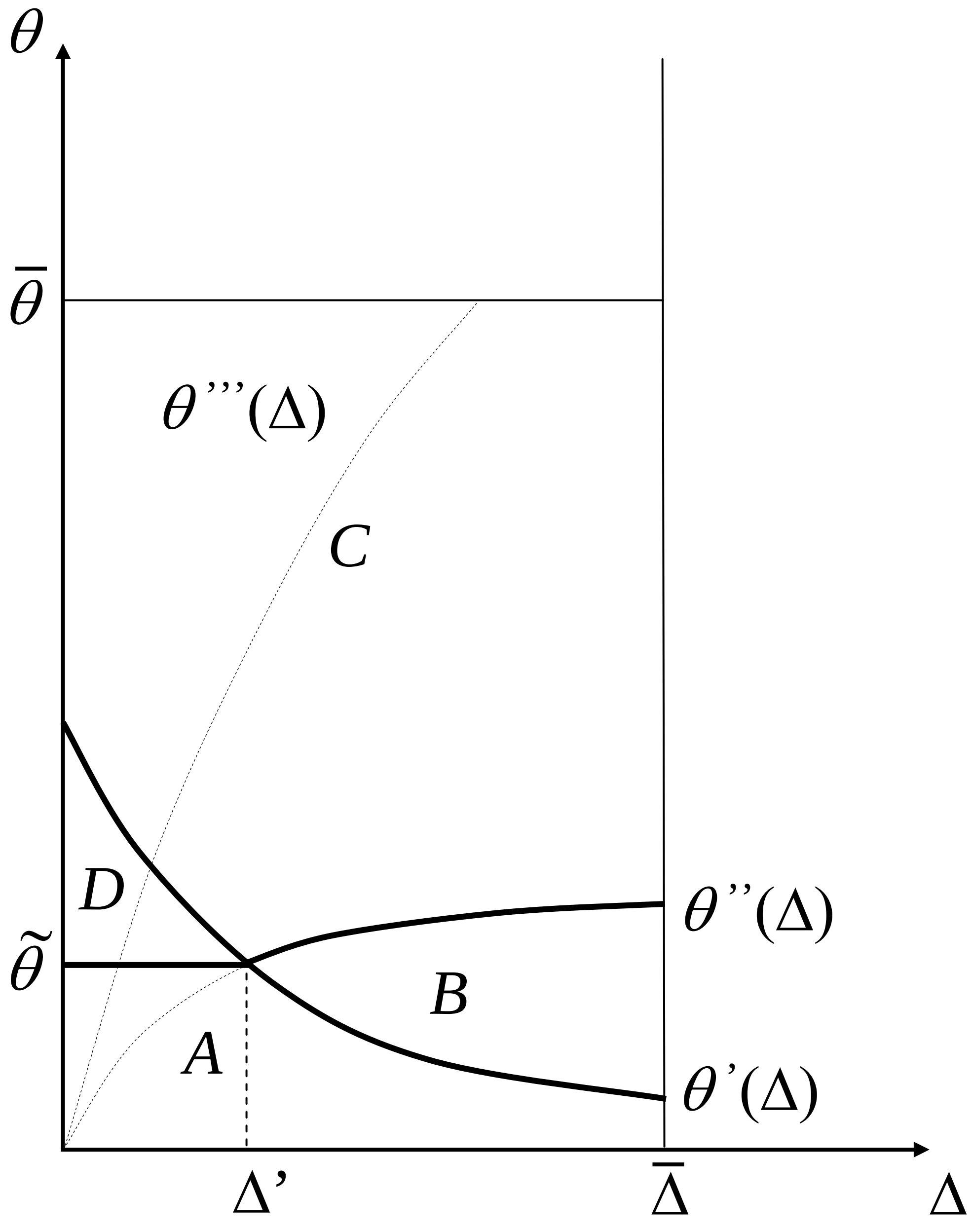


Figure 2. Parameter sets leading to alternative behaviors of the payoffs in the early and intermediate intervals

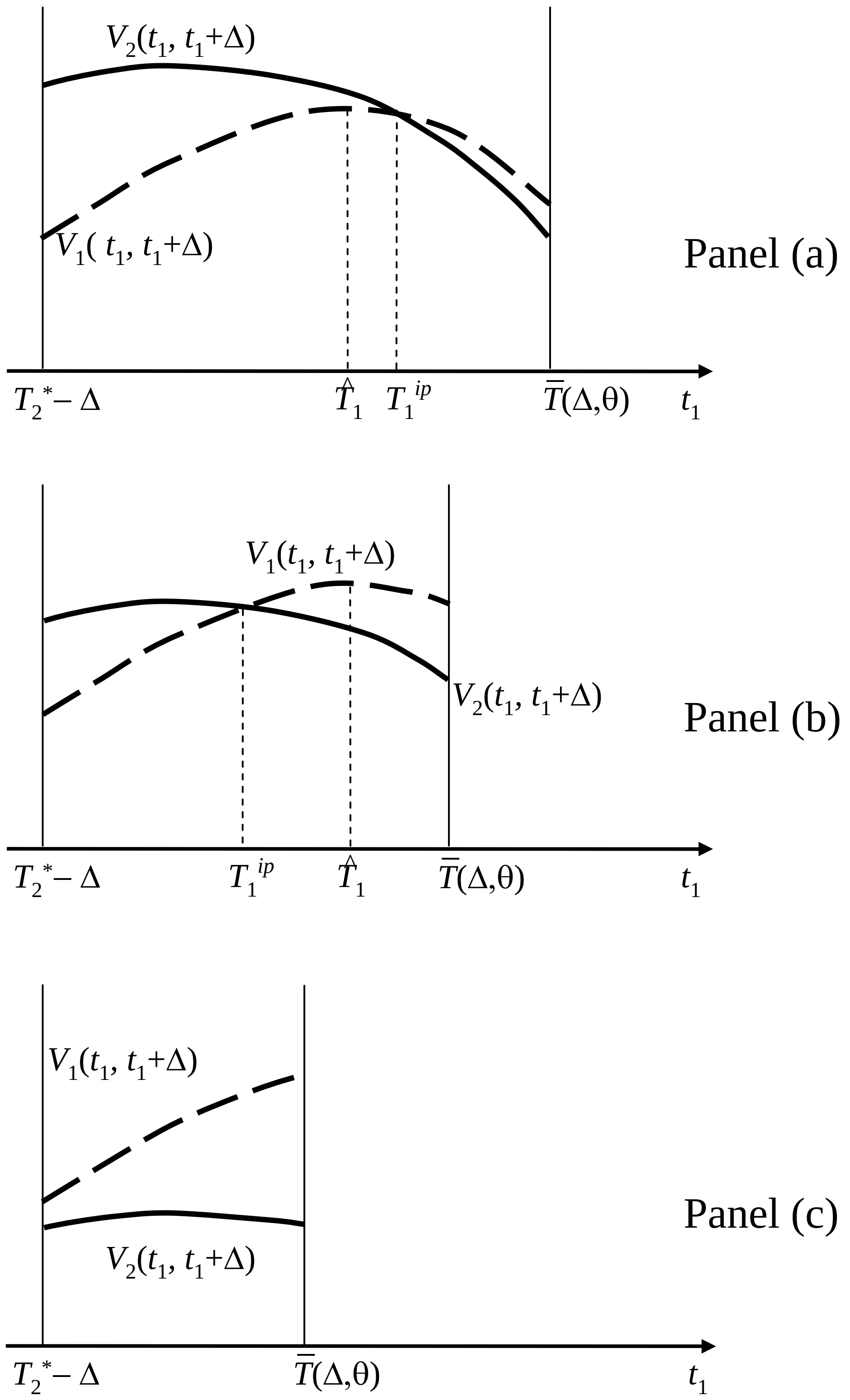
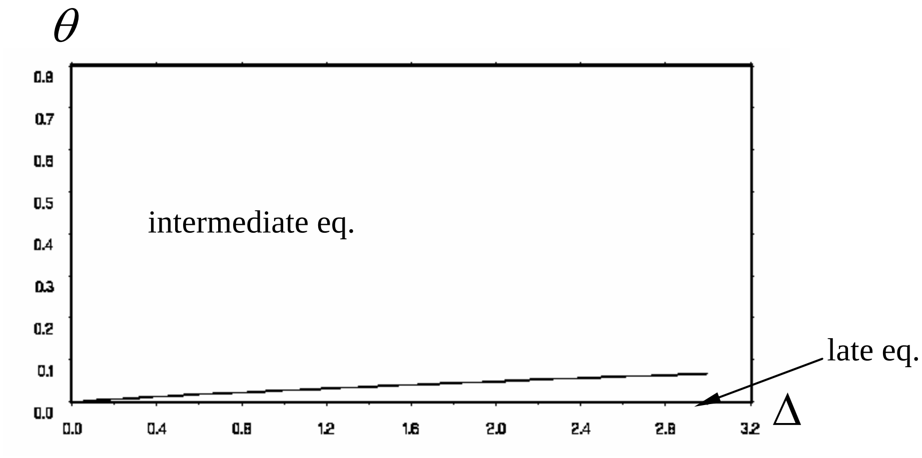
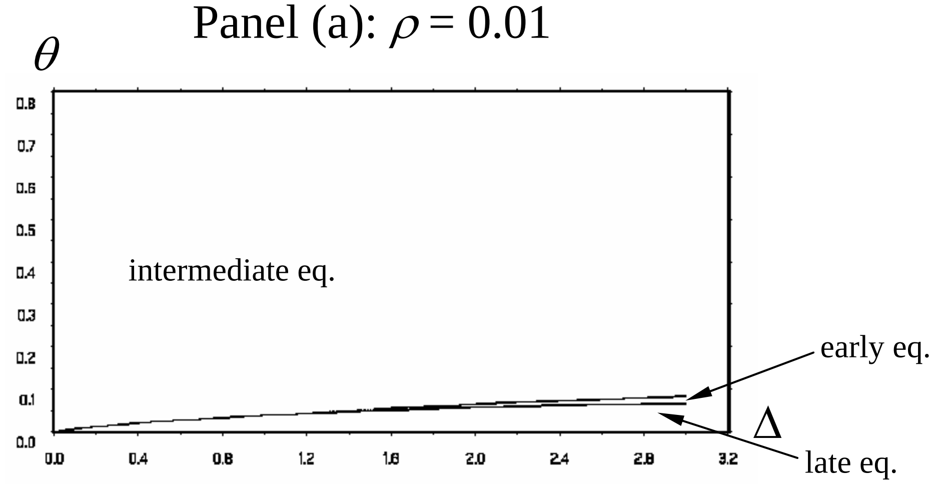


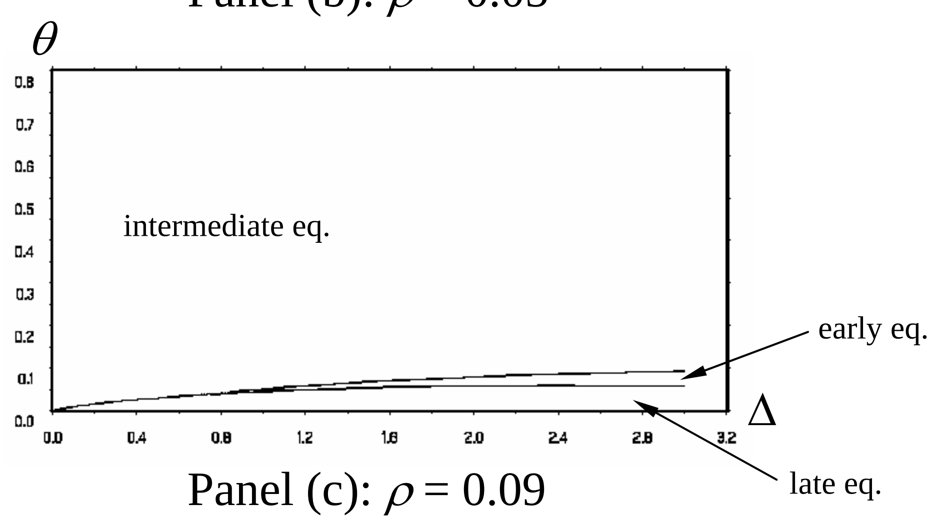
Figure 3: Alternative behaviors for the firms' discounted payoffs in the intermediate interval



Panel (a): $\rho = 0.01$

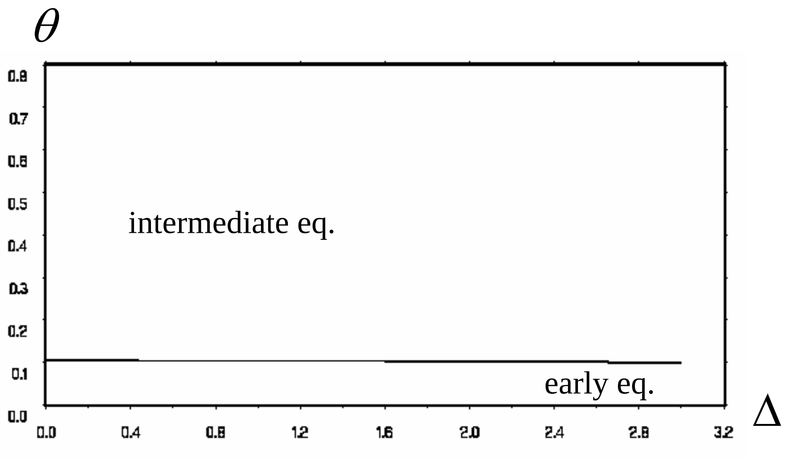


Panel (b): $\rho = 0.05$

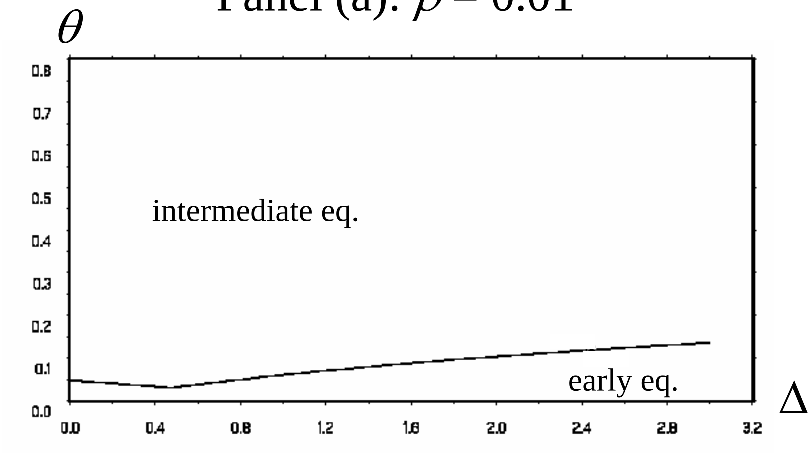


Panel (c): $\rho = 0.09$

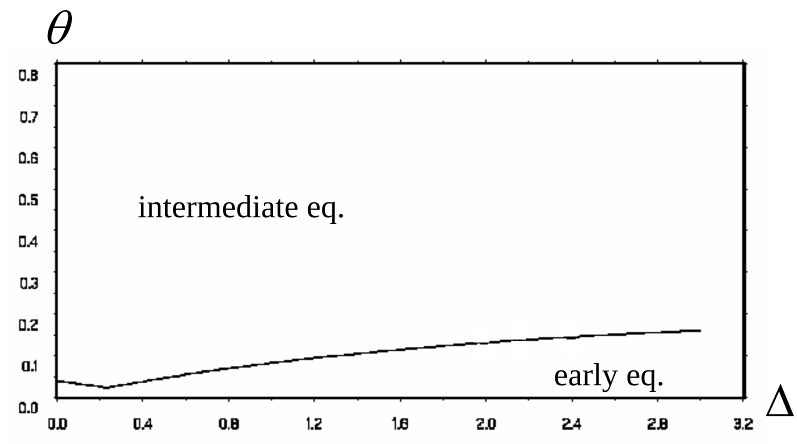
Figure 4 Equilibrium selection – minor innovation ($x = 0.05$)



Panel (a): $\rho = 0.01$



Panel (b): $\rho = 0.05$



Panel (c): $\rho = 0.09$

Figure 5 Equilibrium selection – major innovation (x = 0.50)