

Antitrust policy and price collusion : public agencies vs delegation

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1 Introduction

The ongoing debate on the effectiveness of antitrust policy in limiting anti-competitive activities helps to understand its intrinsic complexity.¹ The perception is that the convicted cases are only a small part of an iceberg of anti-competitive activity.² This implies that the cases to investigate or to supervise are so many that is highly unlikely that an independent public agency in charge of the policy implementation³ may be able to review all of them. As argued by Souam (2001), the antitrust authorities are indeed subject to two sorts of constraints : limited resources (i.e. they cannot deal with all cases and monitor all the markets) and imperfect information⁴ (the

* We wish to thank John D. Hey and Claude d'Aspremont for helpful discussions. We also thank Paolo Buc-cirossi, Giuseppe Colangelo, Gianni De Fraja, Gianluca Femminis, Isabel Grilo and Gerd Weinrich for their comments. We have also greatly benefited from comments and suggestions provided by two anonymous referees. The usual disclaimer applies. The contents expressed in this paper are the authors' view on the issue analyzed and not that of the Italian Antitrust Authority. Correspondence to : G. Martini, Department of Management and Information Technology, University of Bergamo, viale Marconi 5, I-24044 Dalmine (BG), Italy. E-mail : gianmaria.martini@unibg.it

¹ There is still a debate within the economic profession about the social impact and value of antitrust laws. Ross (1984) argued that in many cases antitrust laws were passed in response to political lobbying by small firms complaining about the large firms' ability to secure lower prices in their purchases and supplies.

² For instance, the OECD has recently listed more than 39 industries as worthy of investigation, ranging from shipping to diamonds (see *The Economist*, "Setting the trap", 31st October 2002).

³ Public agency examples are the DG Competition (DGC) (and national authorities) in Europe, the Antitrust Division of the Department of Justice (DOJ) and the Federal Trade Commission (FTC) in the US.

⁴ The diffusion of anti-competitive activities across the industries and the agency's imperfect information imply that in some sectors an investigation is not made because the agency simply does not know that such activities are performed. However a more informed agent (e.g. the consumers or the firms demanding the

agencies do not observe many characteristics and behaviors of the firms).

In this paper we focus on the former constraint and investigate whether it is possible to improve the overall effectiveness of antitrust policy by designing a scheme where different agents may act against anti-competitive activities: a public agency and consumers. If consumers bring a case to Courts we have a regime where the enforcement of the antitrust policy⁵ is *delegated* to private agents, whose preferences may be different from those of the public agency.⁶ If the Court, which is in charge of the final decision under this regime,⁷ identifies an illegal action, consumers get a monetary compensation for their actual damage.⁸

Both the US and European antitrust legislations explicitly declare that private parties may start a case. The US DOJ states that there are three main ways in which the Federal antitrust laws are enforced: criminal and civil enforcement actions brought by itself, civil enforcement actions brought by the FTC and lawsuits brought by private parties asserting damage claims.⁹ The European DGC entitles natural or legal persons to lodge a complaint for violation of antitrust laws.¹⁰

Since our goal is to compare the effectiveness of the public agency and delegation regimes in limiting anti-competitive activities, we design an antitrust game played by an “antitrust” agent and an industry (where firms are potentially engaged in horizontal collusion), in order to identify

good produced by the colluding firms) might instead decide to launch the investigation. The effectiveness of antitrust regimes where private agents and the public agency have asymmetric information about the anti-competitive activity might be the object of future research.

- ⁵ This consists of a decision to carry out an investigation about the firms' behavior, undertaken to establish whether they violated the law or not.
- ⁶ In monetary economics there is by now a comprehensive literature beginning with Rogoff (1985) arguing that a government should delegate monetary policy to a central banker who is independent and more conservative than the government itself. A recent paper by Schultz (1999) shows instead that in presence of polarization, delegation may reduce the incentive to achieve a deflationary policy.
- ⁷ It is essential to point out that analyzing the delegation regime is different from investigating the impact of some new instruments as the *leniency* programs (Motta and Polo (2002)). The latter have been successful in discovering some collusive behaviors because they increase the cartel's members (i.e. those which benefit from the collusion but prefer to reduce the risk of paying a fine) incentive to defect (even if the literature has not considered the reputation effects of reporting to the public agency). The delegation regime regards instead the incentive to launch an investigation of those agents injured by the collusive behavior.
- ⁸ In the US the Clayton Act is a civil statute (carrying no criminal penalties), passed in 1914 and amended in 1950. Title 15, Chapter 1, Section 15 (“Suits by persons injured”) permits private parties injured by an antitrust violation to sue to Federal Court for three times their actual damage, plus Court costs and attorneys' fees.
- ⁹ US DOJ, Antitrust Division, “Antitrust Enforcement and the Consumer”, Section 3, available at www.usdoj.gov/atr/public/div_stats/9142.htm. Salop and White (1988) and Bizjak and Coles (1995) show that in the US the number of cases initiated by private parties is much bigger than those initiated by the DOJ or the FTC.
- ¹⁰ Council Regulation (EC) N. 1/2003, Chapter III, article 7. There is an important difference between the US and European legislation concerning such “private” lawsuits: in the US private agents directly bring the case to Courts and, if they win, get a monetary reward. In Europe, they signal the case to the public agency and do not get any compensation (and this strongly limits private enforcement actions of antitrust laws). Hence the European approach does not relax the public agency's limited resources constraint. This difference is due to the European choice to have a unique subject acting as detective and judge (while these two functions are separated in the US).

the optimal policy and the degree of collusive behavior tolerated (i.e. the maximum level of output not triggering an investigation). The equilibrium outcomes are then ranked in social welfare terms, in order to highlight the regimes' effectiveness.¹¹

Our main finding is that *social welfare is greater if both types of agents can launch an investigation, and so letting consumers to play an active role in fighting anti-competitive activities may reduce the impact of the public agency's limited resources constraint and increase the antitrust policy effectiveness.*

We will show that delegation weakly dominates the public agency regime, and that this result depends upon the differences in the objective functions of the two antitrust agents: consumers only care about their surplus and so they consider the reward they get in presence of anti-competitive activities as an incremental surplus, while the public agency takes into account both consumers and producers surplus and sees the fine as a monetary transfer. As a consequence, *consumers will credibly start off a higher level of investigation activity than that set by a public agency.* The agency, as a result, tolerates a higher degree of collusion than consumers, i.e. the latter will intervene even in presence of "small" violations of the antitrust law, while the former will move only against "relevant" anti-competitive behaviors.

Looking at our results more in details, we have reached the following: First, delegation achieves the first best solution (i.e. marginal costs pricing) in case of complete information. Second, such a solution is not reached if the industry has private information about its production efficiency, and the equilibrium might be either pooling or semi-separating. Third, delegation dominates the public agency regime *independently* of the rule of reimbursement of investigation costs: if the burden of these costs is the same between the two regimes, dominance is still valid. Fourth, the public agency regime never reaches the first best solution. Moreover, if the agency has imperfect information about production costs, the same second best outcome (in terms of social welfare) is achieved under separating equilibria, semi-separating equilibria and the unique pooling equilibrium. Last, under both regimes, the pooling equilibrium is the unique solution where the more efficient industry type enjoys an informational rent.

These results have been reached starting from a unified framework, where there is a unique antitrust agent, and where the two regimes considered here can be classified as polar cases.¹² This framework is useful to point out the factors inducing the different behaviors undertaken by the antitrust agent under the two regimes *in presence of the same collusive behavior.* These differences are well highlighted in the complete information

¹¹ This paper analyzes anti-competitive activities where firms are engaged in price-fixing, and form a cartel acting as a monopolist. The model can be extended to vertical agreements and abuses of dominant position (since the conviction can remove the abuse and so it increases the allocative efficiency). The model cannot deal with mergers and acquisitions.

¹² The introduction of a unified framework is an important suggestion made by an anonymous referee.

case, and so in the imperfect information case only the two specific regimes will be analyzed.

Our analysis is related to previous works on antitrust policy effectiveness. Block, Nold and Sidak (1981), Salant (1987) and Besanko and Spulber (1990) explored whether antitrust policy leads to a welfare improvement (if compared with *laissez faire*) or not, and reached, with different views, a positive answer.¹³ Besanko and Spulber (1989*a*) provided a model where a public agency faces an industry possibly engaged in horizontal collusion and showed that in equilibrium, even if the agency can prove an illegal price fixing behavior, some degree of collusion is tolerated. Souam (1998, 2001) investigated the effects on social welfare of alternative regimes of fines.¹⁴

The last two contributions, the closest ones to our work, present a model with the following sequence of events: the public agency moves first by announcing a schedule of probability of investigation. Having observed it the industry decides whether to collude or not and then the agency implements the ex-ante announced policy. Clearly, the last assumption implies that the agency commits herself to the ex-ante announced policy, i.e. she cannot change her decision after having observed the industry's choice. We believe, on the one hand, that this framework does not reflect the actual implementation of antitrust policy. As highlighted by Banks (1992), such an approach lacks of realism, since in real world antitrust disputes the decision to launch an investigation is taken *after* having observed a signal sent by the industry. On the other hand, it does not capture the idea that firms might have a first mover advantage in the antitrust game. Indeed the promulgation of the laws and the institution of a public agency are *once-and-for-all* decisions, but the policy enforcement is based on a case-by-case approach. In the US the DOJ states some examples of illegal actions (e.g. identical prices, fixing quotas, price changes of equal amount at the same time, identical bids, etc.), but she explicitly declares that these signs are by no means conclusive evidence of collusion.¹⁵ Therefore firms choose their market conducts being aware that these will influence the probability of an investigation. Hence we provide a model with a different sequence of events: the industry moves first by choosing an output level and the antitrust agent (being the public agency or the consumers) chooses after having observed the industry's decision.¹⁶

¹³ Salant pointed out that antitrust policy is welfare neutral because private agents may have the "perverse" incentive to be more damaged today in order to get a higher reward tomorrow (the "treble damage award"). Besanko and Spulber showed that Salant's result does not extend to the asymmetric information case (consumers face uncertainty about the future awards, and so they may lose the incentives to be damaged today).

¹⁴ Building on a specific parameterization he showed that fines related to sales are more efficient, in welfare terms, than fines linked with profits, when rents achievable through collusion are not high.

¹⁵ US DOJ, Antitrust Division, "Antitrust Enforcement and the Consumer", Section 7, available at www.usdoj.gov/atr/public/div_stats/9142.htm.

¹⁶ This approach has already been adopted in the tax compliance literature (Andreoni *et al.* (1998)), in regulation (Banks (1992)) and in law economics (Besanko and Spulber (1989*b*)).

The paper is organized as follows: in Section 2 we present the model and the unified framework for the complete information case. Then we analyze the optimal policy if the industry has private information about its costs. Section 3 presents the public agency regime and Section 4 the delegation regime. The comparison between the regimes is displayed in Section 5, while Section 6 highlights the main conclusions of the paper. In the Appendix we report all the propositions' proofs.

2 The model : a unified framework

Consider a market composed by N identical risk-neutral firms producing a homogeneous product. Total industry output is $q = \sum_{j=1}^N q_j$. For simplicity, let us assume that the cost function is characterized by constant marginal costs and no fixed costs, i.e. $C_j(q_j) = \theta q_j$ ($j = 1, 2, \dots, N$). Firms and the antitrust agent (being the public agency or consumers) know the market demand function $D(q)$.¹⁷

As usual, each firm maximizes its profits, given by $\pi_j(q_1, \dots, q_j, \dots, q_N, \theta) = q_j [P(\sum_j(q_j)) - \theta]$. Firms face the following alternative: (a) not to collude, (b) to collude. Under case (a) they act independently and behave as in Bertrand competition, so that the final industry outcome is the competitive output: q^c . If (b) is chosen, they act as a single player, the industry, and they jointly agree on a level of output $q \leq q^c$. The antitrust agent observes q and decides whether to begin an investigation (action $\{i\}$) or not (action $\{ni\}$).¹⁸ Hence, we define, for each observed q , $\beta(q) \in [0, 1]$ as the probability to launch an investigation; if the latter takes on, we assume that it is possible to uncover whether horizontal collusion has occurred.¹⁹ In this case a fine $F \in [0, A]$, where A (with $A < \infty$) is the maximum fine²⁰, is enforced to the guilty firms, and equally shared among them.²¹ Moreover, some behavioral constraints (*cease and desist* orders and *injunctions*, i.e. specific acts which can directly modify the social welfare) are imposed, so

¹⁷ It is assumed that the cartel supplies the entire market demand. It is possible to adapt the problem to a cartel which acts as a dominant firm and faces a competitive fringe, with residual demand $RD(p) = D(p) - S(p)$, where $S(p)$ is the fringe's supply function.

¹⁸ An investigation can cover both a working agreement and a planned arrangement communicated to the antitrust agent. Under the latter case the cartel "announces" an output level and the public agency decides whether to investigate or not.

¹⁹ Martin (2000) presents a model where there is a probability τ that price-fixing is not detected, and shows that the cartel only cares about the effective probability of being detected, i.e. $\beta(q)(1 - \tau)$.

²⁰ We assume that it is not possible, because of firms' liability, to have a huge fine.

²¹ In principle, the fine should depend upon the damage due to the illegal behavior. In this case we should have a fine $F(q)$, with $\frac{dF(q)}{dq} < 0$. However, an interesting feature of our model is that social welfare under the public agency regime is independent from the fine's magnitude, while under delegation social welfare is increasing in F . Therefore our results hold also in case of endogenous fine.

that firms' coordination becomes unfeasible²² and a Bertrand-Nash solution is achieved.²³ Last, the investigation involves some fixed costs K , with $K > 0$;²⁴ the impact of different rules about the payment of these costs will be analyzed.

The antitrust agent has the following "general" objective function :

$$W = \int_0^q P(x)dx - P(q)q + \lambda[P(q) - \theta]q, \quad (0 \leq \lambda \leq 1) \quad (1)$$

where λ is the weight given to industry profit.²⁵ This is a formulation where the consumer surplus cannot have a lower weight than industry profit, and where the public agency regime is the case where $\lambda = 1$ (the public agency maximizes the social welfare with equal weight on consumer surplus and industry profit), while the delegation regime is the case with $\lambda = 0$ (consumers maximize their surplus). Concerning the payment of investigation costs K , we define ϑ as the fraction of these costs paid by the antitrust agent, with $0 \leq \vartheta \leq 1$. Again the public agency regime is the case where $\vartheta = 1$ (since the agency takes fully into account the policy costs), while under delegation we have that $\vartheta = 1$ only if consumers lose the suit and that $\vartheta = 0$ if instead they win it. From our assumptions, we have the following antitrust agent's payoff (W) if an output $q < q^c$ is observed and a successful investigation is launched :

$$W(i) = \int_0^{q^c} P(x)dx - \theta q + F - \lambda F - \vartheta K = CS(q^c) + (1 - \lambda)F - \vartheta K \quad (2)$$

while if the same output is observed and no investigation is launched, the antitrust agent gets (1), i.e. $W(ni) = CS(q) + \lambda\pi(q)$. Since $\beta(q)$ is the probability that an investigation is launched, the industry's expected profit if they decide to collude is as follows :²⁶

$$\operatorname{argmax}_q (1 - \beta(q))\pi(q) - \beta(q)[F + (1 - \vartheta)K] \quad (3)$$

²² This hypothesis is reinforced by Bizjak and Coles's (1995) empirical results : by studying private antitrust litigation in the US they found that the threat of potentially monetary damages lacks power to explain defendants' behavior. In contrast, the potential imposition of behavioral constraints is their main concern. Hence it seems reasonable to assume that antitrust policy can enforce a more competitive solution since, at least in the short-run, it modifies firms' behavior.

²³ In a static framework, as that depicted in the model, it is also possible to make a less extreme assumption in case of conviction, e.g. a Cournot equilibrium, but this will not change significantly the analysis. Indeed it is essential that an investigation yields a positive effect on the intensity of competition within the industry.

²⁴ These costs may be due, for instance, to administrative costs, legal costs etc. We assume that there are no variable costs of investigation, i.e. those costs affected by the antitrust agent's effort.

²⁵ It is straightforward to show that, if $\lambda \neq 1$, the output maximizing (1) is $q = -\frac{\lambda}{1-2\lambda}(1 - \theta) > q^c$. However, since the industry cannot be forced to produce more than the competitive output, we assume that a feasible solution of the maximization problem (1) belongs to $[0, q^c]$, i.e. we have a corner solution at $q^* = q^c$.

²⁶ We assume that the profit function, $\pi_j(q_1, \dots, q_j, \dots, q_N, \theta)$, is quasi-concave in $q_j \forall j$. Hence there exists a unique monopoly quantity, q^M .

The timing of the antitrust game is the following:²⁷ at $t = 1$ firms decide whether to collude or not, and at $t = 2$ the antitrust agent chooses whether launching an investigation is worthy or not. The policy is implemented and the players' payoffs are computed.

2.1 The complete information case

To derive the optimal policy we look for a *subgame perfect equilibrium* starting, by backward induction, from computing the antitrust agent's optimal strategy at $t = 2$. If q^c is observed, the antitrust agent will choose $\{ni\}$; if instead $q < q^c$, from (2) and $W(ni) = CS(q) + \lambda\pi(q)$, we can state the following Lemma.

Lemma 1 *In case of complete information the antitrust agent's optimal strategy at $t = 2$ is as follows :*

$$\beta(q) = \begin{cases} 1 & \text{if } 0 < q < \tilde{q} \\ 0 & \text{if } \tilde{q} \leq q \leq q^c \end{cases} \quad (4)$$

where \tilde{q} satisfies

$$CS(q^c) + (1 - \lambda)F \geq CS(\tilde{q}) + \lambda\pi(\tilde{q}) + \vartheta K \quad (5)$$

If $\tilde{q} < q^c$, the antitrust agent tolerates some degree of collusion.

Lemma 1 shows that the antitrust policy against horizontal collusion in a specific industry is an interesting problem if there exists an output \tilde{q} higher than q^M ²⁸ where the antitrust agent is indifferent between starting an investigation or doing nothing. Unless $\tilde{q} = q^c$ a sufficiently small degree of collusion is tolerated. Clearly, \tilde{q} is inversely related to ϑ . Given the policy shown in (4) the industry at $t = 1$ has the following strategy :

$$q = \begin{cases} \tilde{q} & \text{if } q^M < \tilde{q} \\ q^M & \text{if } 0 < \tilde{q} \leq q^M \end{cases} \quad (6)$$

²⁷ As mentioned before, one possibility is that the antitrust agent moves first, and selects a schedule of probabilities to launch an investigation (for each $q \in [0, q^c]$), that is announced to the firms. The latter observes it and decide whether to collude or not. Then the antitrust agent implements the previously announced schedule, i.e. the decision to launch an investigation is taken *before* observing the industry's choice. Under this timing it is straightforward to show that the first best solution is always achieved (i.e. independently of the magnitude of λ and ϑ) by announcing the following policy :

$$\beta(q) = \begin{cases} 1 & \text{if } q < q^c \\ 0 & \text{otherwise} \end{cases}$$

so that the public agency regime (where $\lambda = \vartheta = 1$) achieves the first best solution, and a comparison with delegation is uninteresting.

²⁸ If $\tilde{q} < q^M$ then the industry can produce the monopoly output without the threat of being prosecuted for anti-competitive activities.

We can now focus on the following “polar” cases : (1) $\lambda = \vartheta = 1$ (the public agency regime), (2) $\lambda = 1, \vartheta = 0$, (3) $\lambda = 0, \vartheta = 1$ and (4) $\lambda = \vartheta = 0$ (the delegation regime). By comparing the players’ strategies under the above cases we will provide some useful insights about the welfare properties of the antitrust policy and, above all, we will highlight the role played in the two regimes considered by two important factors : (1) the difference in the objective functions (identified by the parameter λ) and (2) the different rules of reimbursement of the investigation costs (i.e. the parameter ϑ).

Under case (1) (i.e. the public agency regime) condition (5) amounts to : $CS(q^c) - CS(\tilde{q}) - \pi(\tilde{q}) \geq K$, so that $\tilde{q} < q^c$ and a small degree of collusion is accepted. Consequently, at $t = 1$ the industry will restrict output at \tilde{q} . Under case (2) we can observe the impact on the policy implemented by a public agency ($\lambda = 1$) of a rule of reimbursement of investigation costs similar to that adopted under the delegation regime ($\vartheta = 0$, if the investigation is successful). Under this settings \tilde{q} is defined by the following condition : $CS(q^c) - CS(\tilde{q}) - \pi(\tilde{q}) \geq 0$. The latter is always fulfilled and so in this case $\tilde{q} = q^c$, i.e. the first best solution is achieved. Case (3) is a situation where the antitrust agent has the same objective function than under the delegation regime ($\lambda = 0$), but it has to bear all the investigation costs ($\vartheta = 1$). Here \tilde{q} is defined implicitly by the following condition : $CS(q^c) - CS(\tilde{q}) + F - K \geq 0$, which is always fulfilled as long as $F > K$. Again $\tilde{q} = q^c$, i.e. the first best solution is achieved. Last, if $\lambda = \vartheta = 0$ (case (4)) we have the delegation regime, and condition (5) becomes $CS(q^c) - CS(q) + F \geq 0$, which is clearly always satisfied, and so $\tilde{q} = q^c$. The analysis points out that the first best solution is achieved in all cases but one, the public agency regime with $\vartheta = 1$, and so we have shown the following :

Proposition 1 *In case of complete information delegation achieves the first best solution and (weakly) dominates the public agency regime. The dominance does not depend on the rules of reimbursement of K .*

The intuition underlying Proposition 1 is the following : the public agency regime is dominated by the delegation regime unless $\vartheta = 0$. However, if $\vartheta = 1$, i.e. also consumers have to pay the investigation costs in case of success as in the public agency regime, the delegation regime achieves the first best solution, while the public agency tolerates a little degree of collusion. It means that if the burden of K is the same between the two regimes and $\vartheta \neq 0$, delegation dominates the public agency regime. The explanation is that consumers, even in the extreme case where $\vartheta = 1$, will choose to launch an investigation if $q < q^c$ as long as $F > K$. Indeed F is a reward that increases consumers surplus, while for the agency is only a monetary transfer. Hence the two regimes yield different policies and achieve different outcomes because of the difference in the antitrust agent’s objective functions, and the rule of reimbursement of the investigation costs does not influence the result.

2.2 The imperfect information case

We now modify the game introducing an asymmetric information structure : firms know their own costs, but the public agency and the consumers do not. Each firm has the following costs function : $C_j(q_j) = \theta_j q_j$ ($j = 1, 2, \dots, N$), with $\theta_i \in \{\theta_1, \theta_2\}$, and $\theta_1 < \theta_2$. All firms are perfectly correlated in efficiency, and we denote γ_1 as the probability that all of them have low marginal cost, and γ_2 as the probability that $\theta = \theta_2$ (with $\gamma_1 + \gamma_2 = 1$). We assume that *prior* probabilities are common knowledge.

Since our aim is to compare the public agency and the delegation regimes, we now focus our attention on the two polar cases (in relation with the general model presented in Section 2.1) where $\lambda = \vartheta = 1$ and $\lambda = \vartheta = 0$, in order to identify the impact of imperfect information on the equilibrium antitrust policies and on the social welfare.

3 Imperfect information : optimal policy under a public agency regime

In this Section we consider the problem faced by a public agency which plays the antitrust game and has imperfect information about the industry's production costs, and with $\lambda = \vartheta = 1$. We define $\mu(\theta_i|q)$ as the agency's *posterior* belief that the industry's efficiency state is θ_i when the output q is observed, with $\mu(\theta_1|q) + \mu(\theta_2|q) = 1, \forall q$. Moreover, let us define α_q^i as the probability that the θ_i ($i = 1, 2$) industry type produces the output q (with $\int_0^{q_i^c} \alpha_q^i dq = 1$). If output q is observed, a *laissez-faire* policy (i.e. no investigation) is associated to the following expected payoff

$$W_q(\beta(q) = 0) = \mu(\theta_1|q) \int_0^q [P(x) - \theta_1] dx + \mu(\theta_2|q) \int_0^q [P(x) - \theta_2] dx \quad (7)$$

If instead the agency investigates at q , her expected payoff is

$$W_q(\beta(q) = 1) = \mu(\theta_1|q) \int_0^{q_1^c} [P(x) - \theta_1] dx + \mu(\theta_2|q) \int_0^{q_2^c} [P(x) - \theta_2] dx - K \quad (8)$$

We denote \tilde{q}_i ($i = 1, 2$) as the output that in case of complete information in each state makes the agency indifferent between investigating and *laissez-faire*. Hence from (5), and with $\lambda = \vartheta = 1$, \tilde{q}_i is defined implicitly as the lowest solution of the following equation :

$$\int_0^{\tilde{q}_i} [P(x) - \theta_i] dx = \int_0^{q_i^c} [P(x) - \theta_i] dx - K \quad (9)$$

that is, $W_i(\tilde{q}_i) = W_i(q_i^c) - K$. Note that $\tilde{q}_i < q_i^c$, and $\tilde{q}_2 < \tilde{q}_1$. We assume that $\tilde{q}_i > q_i^M$, which holds if K is small enough, $q_1^M < \tilde{q}_2$ and $\tilde{q}_1 > q_2^c$.

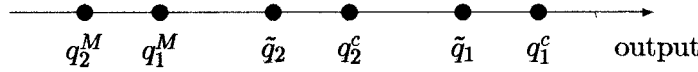


Figure 1 *Output levels*

The latter implies that the efficiency gap is sufficiently large, as one would reasonably expect for imperfect information about industry costs to matter, while $q_1^M < \tilde{q}_2$ means that the costs differential between the two states must not be too large. Figure 1 shows the relative magnitudes of the above outputs.

We look for an *undefeated* equilibrium (Mailath *et al.* (1993)), i.e. a refinement of a *Perfect Bayesian Equilibrium (PBE)*. We define $\sigma \equiv (\alpha^*, \beta^*, \mu^*)$ as a *PBE* if the following conditions hold :

1. $\alpha_q^i \in \operatorname{argmax}_q (1 - \beta^*(q))\pi(q, \theta_i) - F\beta^*(q), (i = 1, 2)$;
2. $\beta^*(q) \in \operatorname{argmax}_\beta \beta(q)\{\sum_{i=1}^2 \mu^*(\theta_i|q)[W_i(q_i^c) - K]\} + (1 - \beta(q))[\sum_{i=1}^2 \mu^*(\theta_i|q)W_i(q)], \forall q$;
3. $\mu^*(\cdot|q)$ satisfies Bayes rule and it is consistent with industry's strategies and prior probabilities.

Next, assume that both $\sigma \equiv (\alpha, \beta, \mu)$ and $\sigma' \equiv (\alpha', \beta', \mu')$ are *PBE*, so that there exist multiple equilibria. With an abuse of notation we write (as in Mailath *et al.* (1993)) $\pi(\sigma, \theta_i)$ for type i 's payoff associated with σ . Then σ *defeats* σ' if $\exists q$ such that :

- $\forall i, \alpha_q^{i'} = 0$ while for some $i \alpha_q^i \neq 0$;
- $\forall i$ such that $\alpha_q^i \neq 0$, then $\pi(\sigma, \theta_i) \geq \pi(\sigma', \theta_i)$ and $\pi(\sigma, \theta_i) > \pi(\sigma', \theta_i)$ for some of them;
- $\mu'(q|\theta_i) = 0$ exactly for those types for which $\alpha_q^i \neq 0$ and $\pi(\sigma, \theta_i) > \pi(\sigma', \theta_i)$.

Hence σ is *undefeated* if there does not exist any σ' that *defeats* σ .²⁹ From the above analysis, we can state the following Lemma, that will be useful in computing the equilibrium :

Lemma 2 *Any equilibrium of the antitrust game under the public agency regime and imperfect information about production costs has the following features :*

- i. *the highest observed output is \tilde{q}_1 , since $\forall q \in]q_2^c, \tilde{q}_1[\rightarrow \beta(q)^* = 1$ and $\forall q \in [\tilde{q}_1, q_1^c] \rightarrow \beta(q)^* = 0$. The lowest output, denoted as q_2 , belongs to the interval $[\tilde{q}_2, q_2^c]$, since $\forall q < \tilde{q}_2 \beta(q)^* = 1$;*
- ii. *$\mu^*(\theta_1|\tilde{q}_1) = 1$, while if $\mu^*(\theta_1|\tilde{q}_2) \neq 0$ then $\beta^*(\tilde{q}_2) = 1$;*

²⁹ The intuition underlying the refinement is the following : consider a proposed *PBE* and a message that is not sent in the equilibrium. Suppose there is an alternative *PBE* equilibrium in which some non-empty set of types of the industry choose the alternative equilibrium to the proposed equilibrium. The test requires the receiver's beliefs at that action in the original equilibrium to be consistent with this set. If the beliefs are not consistent, the second equilibrium defeats the proposed equilibrium.

iii. $\forall q_2 \in [\tilde{q}_2, q_2^c]$, if we define $\beta_i(q_2)$ ($i = 1, 2$) as the probability which makes type θ_1 (θ_2) indifferent between producing q_2 and \tilde{q}_1 (q_2^c), then $\frac{d\beta_i(q_2)}{dq_2} < 0$.

Proof See Appendix.

With the restrictions in the output feasible set displayed in Lemma 2, we can define three types of equilibria in the antitrust game depending upon industry's strategy.

a. *Separating*, where $\alpha_{\tilde{q}_1}^{1*} = 1$ and $\alpha_{q_2^c}^{2*} = 1$ as in the perfect information case, so that $\mu^*(\theta_1|\tilde{q}_1) = \mu^*(\theta_2|\tilde{q}_2) = 1$, $\mu^*(\theta_1|q) = 1 \forall q \neq \tilde{q}_2$, and where the following policy is implemented

$$\beta^*(q) = \begin{cases} 0 & \text{if } \tilde{q}_1 \leq q \leq q_1^c \\ \hat{\beta} & \text{if } q = \tilde{q}_2 \\ 1 & \text{if } q < \tilde{q}_1, q \neq \tilde{q}_2 \end{cases} \quad (10)$$

with

$$\frac{\pi(\tilde{q}_2, \theta_1) - \pi(\tilde{q}_1, \theta_1)}{\pi(\tilde{q}_2, \theta_1) + F} \leq \hat{\beta} \leq \frac{\pi(\tilde{q}_2, \theta_2)}{\pi(\tilde{q}_2, \theta_2) + F}; \quad (11)$$

b. *Pooling* at $q_2 \in]\tilde{q}_2, q_2^c]$, where $\alpha_{q_2^c}^{1*} = \alpha_{q_2^c}^{2*} = 1$, so that $\mu^*(\theta_1|q_2^*) = \gamma_1$, $\mu^*(\theta_2|q_2^*) = \gamma_2$, $\mu^*(\theta_1|q) = 1 \forall q \neq q_2^*$ and where the following policy is implemented

$$\beta^*(q) = \begin{cases} 0 & \text{if } q_2^* \leq q \leq q_1^c \\ 1 & \text{if } 0 \leq q < q_2^* \end{cases}; \quad (12)$$

c. *Semi-separating* at $q_2 \in]\tilde{q}_2, q_2^c]$, where $\alpha_{\tilde{q}_1}^{1*} > 0$, $\alpha_{q_2^c}^{1*} > 0$, $\alpha_{\tilde{q}_1}^{1*} + \alpha_{q_2^c}^{1*} = 1$, $\alpha_{q_2^c}^{2*} = 1$, so that

$$\mu^*(\theta_1|q_2) = \frac{K - W_2(q_2^c) + W_2(q_2)}{W_1(q_1^c) - W_1(q_2) - W_2(q_2^c) + W_2(q_2)} \quad (13)$$

$\mu^*(\theta_1|\tilde{q}_1) = 1$, $\mu^*(\theta_1|q) = 1 \forall q \neq q_2$, and where the following policy is implemented

$$\beta^*(q) = \begin{cases} 0 & \text{if } \tilde{q}_1 \leq q \leq q_1^c \\ \frac{\pi(q_2, \theta_1) - \pi(\tilde{q}_1, \theta_1)}{\pi(q_2, \theta_1) + F} & \text{if } q = q_2 \\ 1 & \text{if } q < \tilde{q}_1 \text{ and } q \neq q_2 \end{cases} \quad (14)$$

We can now state when the above equilibria are enforced by the optimal antitrust policy.

Proposition 2 *Under the public agency regime and imperfect information about production costs the antitrust game may have separating equilibria, a unique undefeated pooling equilibrium and semi-separating equilibria.*

a. Separating equilibria exist iff $A \geq 0$, where

$$A = \pi(\tilde{q}_2, \theta_2) + \pi(\tilde{q}_1, \theta_1) - \pi(\tilde{q}_2, \theta_1); \quad (15)$$

while if $A < 0$ they exist iff

$$F \leq -\frac{\pi(\tilde{q}_2, \theta_2)\pi(\tilde{q}_1, \theta_1)}{A} \quad (16)$$

b. A unique undefeated pooling equilibrium at $q_2^* \in]\tilde{q}_2, q_2^c]$ exists iff q_2^* solves

$$K = \gamma_1[W_1(q_1^c) - W_1(q_2^*)] + \gamma_2[W_2(q_2^c) - W_2(q_2^*)] \quad (17)$$

c. Semi-separating equilibria at $q_2 \in]\tilde{q}_2, q_2^c]$ exist iff $B \geq 0$, where

$$B = \pi(q_2, \theta_2) + \pi(\tilde{q}_1, \theta_1) - \pi(q_2, \theta_1)$$

while if $B < 0$ they exist iff

$$F \leq -\frac{\pi(q_2, \theta_2)\pi(\tilde{q}_1, \theta_1)}{B}, \quad (18)$$

with

$$\alpha_{q_2}^{1*} = \frac{\gamma_2[K - W_2(q_2^c) + W_2(q_2)]}{\gamma_1[W_1(q_1^c) - W_1(q_2) - K]} \quad (19)$$

Proof See Appendix.

Proposition 2 points out the optimal policy under the three equilibrium types. It is interesting to note that no policy is observed under the pooling equilibrium, while the separating and semi-separating equilibria involve some investigation activities. Moreover, note that if condition (17) is not fulfilled $\forall q_2 \in]\tilde{q}_2, q_2^c]$, and if $\mu(\theta_1|q_2) > \frac{K - \mu(\theta_2|q_2)[W_2(q_2^c) - W_2(q_2)]}{W_1(q_1^c) - W_1(q_2)}$ in the same interval, then the pooling equilibrium is at q_2^c , while the semi-separating equilibrium enforces $q_2 = q_2^c$.

Proposition 2 entails an interesting difference between the equilibria obtained here, where the agency moves after the industry, and the approach followed by Besanko and Spulber (1989a) and Souam (1998, 2001), where the public agency commits to an ex-ante announced strategy. In the latter framework the more efficient industry type enjoys an informational rent under imperfect information (in case of complete information it gets zero profits). In our approach the unique equilibrium where the more efficient type enjoys such rent is the pooling one, but the less efficient industry type obtains a lower profit than in the complete information case ($q_2^* > \tilde{q}_2$). In the other two types of equilibria the more efficient type gets the same profit obtained in the complete information case. This difference between the two frameworks is due to the different timing: if the agency moves first there

exists a tradeoff between the social costs due to the investigations needed to deter the more efficient industry type from mimicking the less efficient type production and letting it to produce *less* (but close) than its competitive output (and so it gets an informational rent). In our approach there is no ex-ante deterrence, but only a credible threat to launch an investigation: the more efficient industry type cannot reduce its equilibrium output from the complete information level (i.e. \tilde{q}_1) because the promise (without commitment) not to launch an investigation for $q_2^c < q < \tilde{q}_1$ is not credible. The following Proposition, that states an important result about the equilibria's ranking, partly derives from these arguments.

Proposition 3 *Under the public agency regime and imperfect information about production costs all equilibria are equivalent in social welfare terms.*

Proof: See Appendix.

Since the equilibria are payoff-equivalent in social welfare, the simplest way to write the latter under the public agency regime is $\gamma_1 W_1(\tilde{q}_1) + \gamma_2 W_2(\tilde{q}_2)$, which clearly shows that, at the equilibrium, the fine is neutral in social welfare terms. Proposition 3 highlights an important feature of the approach where antitrust policy is carried out with discretion: since no informational rents are granted to the more efficient industry type (except than in the pooling equilibrium, but with the negative consequence of reducing the less efficient type profits with respect to the complete information case), social welfare is always equal to a linear combination between the two complete information equilibrium outcomes.

4 Imperfect information : optimal policy under delegation

The aim of this Section is to investigate a framework where consumers and firms play the antitrust game (with $\lambda = \vartheta = 0$) and the former have imperfect information about production costs. We define $\nu(\theta_i|q)$ as the consumers' *posterior* belief that the industry's efficiency state is θ_i when q is observed, with $\nu(\theta_1|q) + \nu(\theta_2|q) = 1$. Moreover, let λ_q^i be the probability that the θ_i industry type ($i = 1, 2$) produces q (with $\int_0^{q_i^c} \lambda_q^i dq = 1$), and let $\beta_D(q)$ (D stands for delegation) be the probability that consumers bring the case to the Court if q is observed. Hence if consumers choose a *laissez-faire* policy at $q \neq q_i^c$ they get $\nu(\theta_1|q)CS(q) + \nu(\theta_2|q)CS(q) = CS(q)$, while if they begin an inquiry and win, their payoff is $\nu(\theta_1|q)CS(q_1^c) + \nu(\theta_2|q)CS(q_2^c) + F$. If $\beta_D(q) = 0$, the industry gets $\pi(q, \theta_i)$, if instead $\beta_D(q) = 1$ and $q \neq q_i^c$, its payoff is $-(F + K)$.

We can now state the following :

Lemma 3 *Under delegation and imperfect information about production costs the feasible equilibrium quantities are $\{q_1^c, q_2^c\}$, independently of the magnitude of ϑ .*

Proof See Appendix.

Lemma 3 draws an interesting distinction between the delegation approach and the public agency regime analyzed in the previous Section. There, the highest output produced in equilibrium, i.e. \tilde{q}_1 , is lower than q_1^c , which might instead be produced under delegation also in presence of imperfect information. The explanation is the following: since consumers get a reward in case of successful investigation (i.e. at any $q \neq q_i^c$), if this is greater than the investigation costs that they have to pay (and this is always true if $\vartheta = 0$ as in the delegation regime considered here), to intervene is a dominant strategy. As in Section 2, the difference in the objective function among consumers and the public agency leads to different policy outcomes.

To be able to make a full rank between the two regimes, it is necessary to compute the players' equilibrium strategies. In doing so we analyze the players' payoffs at $\{q_1^c, q_2^c\}$ in more details. If q_1^c is observed, consumers' best reply is $\beta_D(q_1^c) = 0$, and they get $CS(q_1^c)$. If instead q_2^c is observed and consumers select $\beta_D(q_2^c) = 1$, their payoff is $\nu(\theta_1|q_2^c)[CS(q_1^c) + F] + \nu(\theta_2|q_2^c)[CS(q_2^c) - K]$, while with $\beta_D(q_2^c) = 0$ their payoff is $CS(q_2^c)$. Hence $\beta_D(q_2^c)$, consumers' best response correspondence at q_2^c , is

$$\beta_D(q_2^c) = \begin{cases} 1 & \text{if } \nu(\theta_1|q_2^c) > \frac{\nu(\theta_2|q_2^c)K}{CS(q_1^c) - CS(q_2^c) + F} \\ 0 & \text{if } \nu(\theta_1|q_2^c) < \frac{\nu(\theta_2|q_2^c)K}{CS(q_1^c) - CS(q_2^c) + F} \\ [0, 1] & \text{if } \nu(\theta_1|q_2^c) = \frac{\nu(\theta_2|q_2^c)K}{CS(q_1^c) - CS(q_2^c) + F} \end{cases} \quad (20)$$

Moreover $\lambda_{q_2^c}^1$, since the θ_1 industry type's expected profit at q_2^c is $(1 - \beta_D(q_2^c))\pi(q_2^c, \theta_1) - \beta_D(q_2^c)(K + F)$ and $\pi(q_1^c, \theta_1) = 0$, is defined as follows

$$\lambda_{q_2^c}^1 = \begin{cases} 1 & \text{if } \beta_D(q_2^c) < \frac{\pi(q_2^c, \theta_1)}{\pi(q_2^c, \theta_1) + K + F} \\ 0 & \text{if } \beta_D(q_2^c) > \frac{\pi(q_2^c, \theta_1)}{\pi(q_2^c, \theta_1) + K + F} \\ [0, 1] & \text{if } \beta_D(q_2^c) = \frac{\pi(q_2^c, \theta_1)}{\pi(q_2^c, \theta_1) + K + F} \end{cases} \quad (21)$$

while $\lambda_{q_2^c}^{2*} = 1$ (the θ_2 industry type makes zero profit). We can now define the two types of equilibria arising in the antitrust game under the delegation regime depending upon industry's strategy:³⁰

- a. *Pooling* at q_2^c , where $\lambda_{q_2^c}^{1*} = \lambda_{q_2^c}^{2*} = 1$, so that $\nu^*(\theta_1|q_2^c) = \gamma_1$, $\nu^*(\theta_2|q_2^c) = \gamma_2$ and where the following policy is implemented

$$\beta_D^*(q) = \begin{cases} 0 & \text{if } q = q_i^c \\ 1 & \text{otherwise} \end{cases} \quad (22)$$

³⁰ A separating equilibrium where $\lambda_{q_1^c}^1 = \lambda_{q_2^c}^2 = 1$ does not exist under delegation, since consumers would always set $\beta_D(q_2^c) = 0$ and so the industry would react by setting $\lambda_{q_2^c}^1 = 1$, so that $\beta_D(q_2^c) = 0$ is not a best reply.

- b. *Semi-separating* at q_2^c , where $\lambda_{q_1^c}^{1*} > 0$, $\lambda_{q_2^c}^{1*} > 0$, $\lambda_{q_1^c}^{1*} + \lambda_{q_2^c}^{1*} = 1$, $\lambda_{q_2^c}^{2*} = 1$, so that $\nu^*(\theta_1|q_1^c) = 1$, $\nu^*(\theta_2|q_1^c) = 0$,

$$\nu^*(\theta_1|q_2^c) = \frac{\gamma_1 \alpha_{q_2^c}^{1*}}{\gamma_1 \alpha_{q_2^c}^{1*} + \gamma_2} = \frac{K}{CS(q_1^c) - CS(q_2^c) + F + K} \quad (23)$$

$$\nu^*(\theta_2|q_2^c) = \frac{\gamma_2}{\gamma_1 \alpha_{q_2^c}^{1*} + \gamma_2} = \frac{CS(q_1^c) - CS(q_2^c) + F}{CS(q_1^c) - CS(q_2^c) + F + K} \quad (24)$$

$\nu^*(\theta_1|q) = 1 \forall q \neq \{q_1^c, q_2^c\}$, and where the following policy is implemented

$$\beta_D^*(q) = \begin{cases} 0 & \text{if } q = q_1^c \\ 1 & \text{if } q_2^c < q < q_1^c \text{ and } q < q_2^c \\ \frac{\pi(q_2^c, \theta_1)}{\pi(q_2^c, \theta_1) + K + F} & \text{if } q = q_2^c \end{cases} \quad (25)$$

The following proposition characterizes the equilibrium under delegation.

Proposition 4 *Under delegation and imperfect information about production costs there exist two mutually exclusive equilibria : a unique pooling equilibrium and a unique semi-separating equilibrium. In both cases the first best solution is no longer achieved.*

- a. A unique pooling equilibrium at q_2^c exists iff

$$K > \frac{\gamma_1}{\gamma_2} [CS(q_1^c) - CS(q_2^c) + F] \quad (26)$$

- b. A unique semi-separating equilibrium at q_2^c exists iff condition (26) does not hold, with

$$\lambda_{q_2^c}^{1*} = \frac{\gamma_2 K}{\gamma_1 [CS(q_1^c) - CS(q_2^c) + F]} \quad (27)$$

Proof See Appendix.

In case of imperfect information the delegation regime cannot reach the first best solution, as instead it does in case of complete information. Indeed consumers, due to imperfect information about industry costs, are no longer sure that output q_2^c is produced only by the less efficient industry type. The more efficient industry type can cheat, producing q_2^c instead than q_1^c . In case of pooling equilibrium consumers cannot avoid this; in the semi-separating equilibrium consumers, to deter the more efficient industry type from *always* producing q_2^c , have to investigate at that output even if there is a chance of unsuccessful investigation. The two equilibria shown in Proposition 4 are mutually exclusive, since the existing one depends upon an exogenous parameter, i.e. K . In order to rank the two regimes, we must

compute the social welfare under the delegation regime. In case of a pooling equilibrium, the expected social welfare is $EW = \gamma_1 W_1(q_2^c) + \gamma_2 W_2(q_2^c)$, while in a semi-separating equilibrium it is

$$EW = \gamma_1(1 - \lambda_{q_2^c}^1)W_1(q_1^c) + \gamma_1 \lambda_{q_2^c}^1 [\beta_D(q_2^c)(W_1(q_1^c) - K) + (1 - \beta_D(q_2^c))W_1(q_2^c)] \\ + \gamma_2(W_2(q_2^c) - \beta_D(q_2^c)K) \quad (28)$$

and by substituting (27) for $\lambda_{q_2^c}^1$ and (25) for $\beta_D(q_2^c)$ we get (by considering that $W_1(q_2^c) = W_2(q_2^c) + \pi(q_2^c, \theta_1)$)

$$EW = \gamma_1 W_1(q_1^c) + \gamma_2 W_2(q_2^c) - \frac{\gamma_2 K [W_1(q_1^c) - W_2(q_2^c)]}{W_1(q_1^c) - W_2(q_2^c) + F} \quad (29)$$

Clearly, social welfare in the semi-separating equilibrium is increasing in F .

5 Comparison between regimes

In this Section we investigate the different performances in social welfare terms of the two regimes. In Section 2 we have already shown that the delegation regime (weakly) dominates the public agency regime in case of complete information. Section 3 and Section 4 have shown that under imperfect information the two investigated regimes yield several equilibria. In the public agency regime the equilibria are all equivalent in welfare terms, that is $EW^P = \gamma_1 W_1(\tilde{q}_1) + \gamma_2 W_2(\tilde{q}_2)$ (where P stands for public agency), while under delegation the two equilibria are mutually exclusive and produce two different levels of expected welfare (defined as EW^D , where D is for delegation). The following Proposition ranks the two regimes in case of imperfect information.

Proposition 5 *If the industry has private information about production costs, the delegation regime dominates the public agency regime in social welfare terms.*

Proof See Appendix.

Note that EW^P is fine neutral, while $\frac{\partial EW^D}{\partial F} > 0$ in case of semi-separating equilibrium³¹ and $\frac{\partial EW^D}{\partial F} = 0$ in case of pooling equilibrium. Hence Proposition 5 is valid independently of the fine magnitude and on its (potentially) different impact under the two regimes.

Propositions 1-5 have shown that both regimes will launch an investigation in case of “relevant” anti-competitive activities (i.e. those involving a social loss higher than the policy costs to fight horizontal collusion), while

³¹ If $F = 0$ expression (A.24) in the Appendix is still greater than 0.

in presence of “minor” violations only consumers have sufficient incentive to launch an investigation. Delegation to consumers is not needed in case of relevant anti-competitive activities (the public agency is effective in fighting them), while society has to rely on consumers to deter minor collusive agreements (since the public agency will instead not intervene). The limited resources constraint leads the public agency to intervene only against the more important cases, and consumers may partially (in case of imperfect information) relax this constraint. Hence our policy recommendation is to explicitly declare in the antitrust legislations that there are two main ways in which antitrust laws are enforced: enforcement actions brought by the public agency and lawsuits brought by private parties asserting damage claims.

6 Conclusions

We have analyzed whether a social design where the antitrust policy is carried out in a complementary way by a public agency and by private agents (namely consumers or their associations) can improve its overall effectiveness. The two agents have different preferences towards the policy outcome: consumers only care about their surplus, the public agency looks at social welfare and takes into account the costs of policy implementation. To consider a delegation regime is important because the action of a public agency is curbed by a limited resources constraint, and so she may choose not to act against “minor” violations of antitrust laws. The optimal policy has been identified in a model where, as in real world antitrust litigation, the antitrust agent (being the public agency or the consumers) carries out its strategy with discretion, i.e. the choice to investigate or not is taken *after* observing a signal sent by the industry. We have compared the two regimes both under complete information and imperfect information about production costs, and we have ranked them in social welfare terms.

The general conclusions that we have achieved are the following: First, delegation may indeed relax the limited resources constraint and so it increases the effectiveness of antitrust policies because consumers will credibly start off a higher level of investigation activity than that set by the public agency. Hence while delegation is not needed to fight against the more important cases (the public agency will credibly act against them because their damages are greater than the investigation costs), it is instead essential to deter the industry from being engaged in “minor” violations (where the public agency will not intervene). This result does not depend upon the rule adopted to reimburse the investigation costs (delegation dominates the public agency even if consumers have to pay all the investigation costs) and it is explained by the difference in the two antitrust agents’ objective function, as in the traditional delegation literature (Rogoff (1985)).

Second, under complete information, delegation completely deters the industry from colluding, while the public agency tolerates a little degree of collusion unless she has not to pay for the investigation. Third, under imperfect information about production costs, complete deterrence is no longer achievable also with delegation (but the latter still dominates the public agency regime). Last, under both regimes the more efficient industry type can enjoy an informational rent only in case of pooling equilibrium (but the less efficient type gets a profits reduction with respect to the complete information case).

The delegation regime considered here seems to be very effective against horizontal collusion. Indeed the model does not take into account of some circumstances that may decrease its enforcement power. For instance, since consumers are too dispersed as a constituency, the model applies well to their associations, while it is less robust if we consider the behavior of many individual consumers. On the contrary, consumers may increase the effectiveness of antitrust policy because they have better information about the industry anti-competitive activities than the public agency. The analysis of these issues is left to future research.

Appendix

Proof of Lemma 2

(i). In case of complete information $\beta^*(q) = 0$ if $q \geq \tilde{q}_1$. Hence, since $\tilde{q}_1 > q_1^M$, $\pi(\tilde{q}_1, \theta_1) > \pi(q, \theta_1) \forall q > \tilde{q}_1$, and so \tilde{q}_1 dominates all $q > \tilde{q}_1$ for the θ_1 industry type. Similarly, in case of complete information, $\beta^*(q) = 1 \forall q < \tilde{q}_2$ even in the worst state of nature, θ_2 . However, in contrast with the perfect information case, the agency may have an incentive to investigate if $q_2 \in [\tilde{q}_2, q_2^c]$, if $\mu(\theta_1|q_2) \neq 0$.

(ii). Note that $\alpha_q^2 = 0 \forall q > q_2^c$ since profits are negative for type θ_2 . Hence $\mu^*(\theta_1|\tilde{q}_1) = 1$. If \tilde{q}_2 is observed and $\beta(\tilde{q}_2) = 0$, the agency's payoff is

$$EW(\beta(\tilde{q}_2) = 0) = \mu(\theta_1|\tilde{q}_2)W_1(\tilde{q}_2) + \mu(\theta_2|\tilde{q}_2)W_2(\tilde{q}_2) \quad (A.1)$$

while if $\beta(\tilde{q}_2) = 1$ she gets

$$EW(\beta(\tilde{q}_2) = 1) = \mu(\theta_1|\tilde{q}_2)[W_1(q_1^c) - K] + \mu(\theta_2|\tilde{q}_2)[W_2(q_2^c) - K] \quad (A.2)$$

Now $EW(\beta(\tilde{q}_2) = 0) < EW(\beta(\tilde{q}_2) = 1)$ when (since $W_2(\tilde{q}_2) = W_2(q_2^c) - K$ and $K = W_1(q_1^c) - W_1(\tilde{q}_1)$)

$$\mu(\theta_1|\tilde{q}_2)[W_1(\tilde{q}_1) - W_1(\tilde{q}_2)] > 0 \quad (A.3)$$

which is always satisfied unless $\mu(\theta_1|\tilde{q}_2) = 0$.

(iii). The probability which makes the θ_1 industry type indifferent between producing q_2 or \tilde{q}_1 is given by

$$\beta_1(q_2) = \frac{\pi(q_2, \theta_1) - \pi(\tilde{q}_1, \theta_1)}{\pi(q_2, \theta_1) + F} \tag{A.4}$$

and if we differentiate it with respect to q_2 , we have

$$\frac{\partial \beta_1(q_2)}{\partial q_2} = \frac{\frac{\partial \pi(q_2, \theta_1)}{\partial q_2} [F + \pi(\tilde{q}_1, \theta_1)]}{[\pi(q_2, \theta_1) + F]^2} \tag{A.5}$$

and since $q_1^M < q_2$, $\rightarrow \frac{\partial \pi(q_2, \theta_1)}{\partial q_2} < 0$ and so $\frac{\partial \beta_1(q_2)}{\partial q_2} < 0$. Hence the lower is $q_2 \in [\tilde{q}_2, q_2^c]$ the higher must be $\beta_1(q_2)$. The same result holds for the θ_2 industry type (indifference between q_2 and q_2^c).

□

Proof of Proposition 2

a. From Lemma 2 $\beta(\tilde{q}_2) = 1$ unless $\mu(\theta_1|\tilde{q}_2) = 0$. Hence to enforce $\alpha_{q_2}^1 = 0$ it is necessary that

$$(1 - \beta(\tilde{q}_2))\pi(\tilde{q}_2, \theta_1) - \beta(\tilde{q}_2)F \leq \pi(\tilde{q}_1, \theta_1) \tag{A.6}$$

and to have $\alpha_{q_2}^2 = 1$ we need

$$(1 - \beta(\tilde{q}_2))\pi(\tilde{q}_2, \theta_2) - \beta(\tilde{q}_2)F \geq 0 \tag{A.7}$$

with $\beta(\tilde{q}_2)$ defined in (A.6) lower than that sufficient to satisfy condition (A.7). Hence if we solve the above conditions for $\beta(\tilde{q}_2)$ and then for F , we have

$$\frac{\pi(\tilde{q}_2, \theta_1) - \pi(\tilde{q}_1, \theta_1)}{\pi(\tilde{q}_2, \theta_1) + F} \leq \hat{\beta} \leq \frac{\pi(\tilde{q}_2, \theta_2)}{\pi(\tilde{q}_2, \theta_2) + F};$$

Rearranging it we get

$$F[\pi(\tilde{q}_2, \theta_2) + \pi(\tilde{q}_1, \theta_1) - \pi(\tilde{q}_2, \theta_1)] \geq -\pi(\tilde{q}_2, \theta_2)\pi(\tilde{q}_1, \theta_1)$$

Setting $A = \pi(\tilde{q}_2, \theta_2) + \pi(\tilde{q}_1, \theta_1) - \pi(\tilde{q}_2, \theta_1)$ leads to three cases :

1. $A > 0$, so that

$$F \geq \underbrace{-\frac{\pi(\tilde{q}_2, \theta_2)\pi(\tilde{q}_1, \theta_1)}{A}}_{<0}$$

that is always fulfilled since $F \gg 0$, so that separating equilibria always exist.

2. $A = 0$, $\rightarrow 0 \geq -\pi(\tilde{q}_2, \theta_2)\pi(\tilde{q}_1, \theta_1)$, that is again always satisfied.

3. $A < 0$, \rightarrow to have a separating equilibrium condition (16) must be fulfilled.

b. From Lemma 2 a pooling equilibrium at \tilde{q}_2 is not possible. $\forall q_2 \in]\tilde{q}_2, q_2^c]$, if $\beta(q_2) = 1$, social welfare is

$$EW(\beta(q_2) = 1) = \gamma_1[W_1(q_1^c) - K] + \gamma_2[W_2(q_2^c) - K] \quad (A.8)$$

while with *laissez-faire* her payoff is

$$EW(\beta(q_2) = 0) = \gamma_1 W_1(q_2) + \gamma_2 W_2(q_2) \quad (A.9)$$

Hence to have a pooling at q_2 it is necessary that (A.9) \geq (A.8). Now let q_2^* be the solution of $K = \gamma_1[W_1(q_1^c) - W_1(q_2)] + \gamma_2[W_2(q_2^c) - W_2(q_2)]$; then all output levels in the interval $[q_2^*, q_2^c]$ can be sustained in a pooling *PBE*. However, only the pooling equilibrium at $q_2 = q_2^*$ is undefeated. To see this, consider *PBE*₁, an equilibrium where, with $\hat{q} > q_2^*$, both types produce \hat{q} , and

$$\beta(q) = \begin{cases} 0 & \text{if } \tilde{q}_1 \leq q \leq q_1^c \text{ or } q = \hat{q} \\ 1 & \text{otherwise} \end{cases}$$

and if $q \neq \hat{q}$ then $\mu^*(\theta_1|q) = 1$. Moreover, consider a second equilibrium, *PBE*₂, such that both types produce q_2^* and

$$\beta(q) = \begin{cases} 0 & \text{if } \tilde{q}_1 \leq q \leq q_1^c \text{ or } q = q_2^* \\ 1 & \text{otherwise} \end{cases}$$

and with if $q \neq q_2^*$ then $\mu^*(\theta_1|q) = 1$. Now suppose that the candidate equilibrium is *PBE*₁ and that q_2^* is observed. Then, according to the out-of-equilibrium beliefs, q_2^* should be produced by type θ_1 . However, in *PBE*₂ type θ_2 produces q_2^* and gets an higher profit than in *PBE*₁. Hence type θ_2 should produce it and the belief that q_2^* should be produced only by type θ_1 is not consistent, so that *PBE*₁ is defeated by *PBE*₂. Vice versa, *PBE*₂ is undefeated, since each type gets a lower profits by producing any $q > q_2^*$.

c. $\forall q_2 \in]\tilde{q}_2, q_2^c]$ if $\beta(q_2) = 1$, social welfare is

$$EW(\beta(q_2) = 1) = \mu(\theta_1|q_2)[W_1(q_1^c) - K] + \mu(\theta_2|q_2)[W_2(q_2^c) - K] \quad (A.10)$$

while, with the alternative move the agency gets

$$EW(\beta(q_2) = 0) = \mu(\theta_1|q_2)W_1(q_2) + \mu(\theta_2|q_2)W_2(q_2) \quad (A.11)$$

Hence $EW(\beta(q_2) = 1) = EW(\beta(q_2) = 0)$ when

$$\mu(\theta_1|q_2) = \frac{K - \mu(\theta_2|q_2)[W_2(q_2^c) - W_2(q_2)]}{W_1(q_1^c) - W_1(q_2)} \quad (A.12)$$

Since, in a semi-separating equilibrium, $\alpha_{q_2}^{2*} = 1$, we have

$$\mu(\theta_1|q_2) = \frac{\alpha_{q_2}^1 \gamma_1}{\alpha_{q_2}^1 \gamma_1 + \gamma_2} \quad (A.13)$$

Substituting (A.13) in (A.12) and solving for $\alpha_{q_2}^1$, we get (19). However, to have $0 < \alpha_{q_2}^1 < 1$, the θ_1 industry type must be indifferent between producing \tilde{q}_1 and q_2 , while to have $\alpha_{q_2}^2 = 1$ we need that producing q_2 yields non-negative profits for type θ_2 . Hence

$$(1 - \beta(q_2))\pi(q_2, \theta_1) - \beta(q_2)F = \pi(\tilde{q}_1, \theta_1) \quad (A.14)$$

and

$$(1 - \beta(q_2))\pi(q_2, \theta_2) - \beta(q_2)F \geq 0 \quad (A.15)$$

Solving (A.14) and (A.15) for $\beta(q_2)$ and rearranging we get

$$F[\pi(q_2, \theta_2) + \pi(\tilde{q}_1, \theta_1) - \pi(q_2, \theta_1)] \geq -\pi(q_2, \theta_2)\pi(\tilde{q}_1, \theta_1)$$

If $B = \pi(q_2, \theta_2) + \pi(\tilde{q}_1, \theta_1) - \pi(q_2, \theta_1)$, then, by repeating the same procedure shown in part *a* of this proof, we have that if $B \geq 0$ a semi-separating equilibrium always exists, while if $B < 0$ condition (18) must be fulfilled. □

Proof of Proposition 3

The separating equilibrium yields the following expected welfare

$$EW_1 = \gamma_1 W_1(\tilde{q}_1) + \gamma_2 \{\beta(\tilde{q}_2)[W_2(q_2^c) - K] + (1 - \beta(\tilde{q}_2))W_2(\tilde{q}_2)\} \quad (A.16)$$

i.e. $EW_1 = \gamma_1 W_1(\tilde{q}_1) + \gamma_2 W_2(\tilde{q}_2)$, since $W_2(q_2^c) - K - W_2(\tilde{q}_2) = 0$ by (9). The expected social welfare under the semi-separating equilibrium is

$$EW_2 = \gamma_1 \{(1 - \alpha_{q_2}^1)W_1(\tilde{q}_1) + \alpha_{q_2}^1 [\beta(q_2)(W_1(q_1^c) - K) + (1 - \beta(q_2))W_1(q_2)]\} \\ + \gamma_2 [\beta(q_2)(W_2(q_2^c) - K) + (1 - \beta(q_2))W_2(q_2)] \quad (A.17)$$

rearranged as (since $W_1(q_1^c) - K = W_1(\tilde{q}_1)$)

$$EW_2 = \gamma_1 \{W_1(\tilde{q}_1) - \alpha_{q_2}^1 (1 - \beta(q_2))[W_1(\tilde{q}_1) - W_1(q_2)]\} + \gamma_2 W_2(q_2) \\ + \gamma_2 \beta(q_2)[W_2(q_2^c) - K - W_2(q_2)] \quad (A.18)$$

so that, by substituting for $\alpha_{q_2}^1$ as defined in (19), we have

$$EW_2 = \gamma_1 W_1(\tilde{q}_1) - \gamma_2 K + \gamma_2 W_2(q_2^c) - \gamma_2 W_2(q_2) + \gamma_2 \beta(q_2)K - \gamma_2 \beta(q_2)W_2(q_2^c) \\ + \gamma_2 \beta(q_2)W_2(q_2) + \gamma_2 W_2(q_2) + \gamma_2 \beta(q_2)W_2(q_2^c) - \gamma_2 \beta(q_2)W_2(q_2) - \gamma_2 \beta(q_2)K \quad (A.19)$$

i.e. $\gamma_1 W_1(\tilde{q}_1) + \gamma_2 W_2(\tilde{q}_2)$, since $W_2(q_2^c) - K = W_2(\tilde{q}_2)$. Hence $EW_1 = EW_2$. Last social welfare under pooling equilibrium is

$$EW_3 = \gamma_1 W_1(q_2^*) + \gamma_2 W_2(q_2^*). \text{ But from (17) we know that} \\ \gamma_1 W_1(q_2^*) + \gamma_2 W_2(q_2^*) = \gamma_1 [W_1(q_1^c) - K] + \gamma_2 [W_2(q_2^c) - K], \text{ and so,} \\ EW_3 = EW_2 = EW_1. \quad \square$$

Proof of Lemma 3

First consider the case $\vartheta = 0$. For any $q < q_2^c$, consumers get, by choosing $\{i\}$:

$$\nu(\theta_1|q)CS(q_1^c) + \nu(\theta_2|q)CS(q_2^c) + F$$

while if they select $\{ni\}$ their payoff is : $CS(q)$. Hence $\{i\}$ dominates $\{ni\}$ when

$$\underbrace{\nu(\theta_1|q)CS(q_1^c) + \nu(\theta_2|q)CS(q_2^c) - CS(q)}_{>0} + F > 0$$

which is always fulfilled. For any $q_2^c < q < q_1^c$ consumers posterior belief is $\nu(\theta_1|q) = 1$, and so $\{i\}$ yields $CS(q_1^c) + F$, while $\{ni\}$ gives $CS(q)$. Again the former dominates the latter.

Then consider the opposite case where $\vartheta = 1$. For any $q < q_2^c$ the consumers' payoff associated to $\{i\}$ is :

$$\nu(\theta_1|q)CS(q_1^c) + \nu(\theta_2|q)CS(q_2^c) + F - K$$

while $\{ni\}$ gives $CS(q)$. Hence $\{i\}$ dominates $\{ni\}$ when

$$\underbrace{\nu(\theta_1|q)CS(q_1^c) + \nu(\theta_2|q)CS(q_2^c) - CS(q)}_{>0} + F - K > 0$$

which is always fulfilled if $F > K$. If instead $q_2^c < q < q_1^c$ then $\nu(\theta_1|q) = 1$ and $\{i\}$ yields $CS(q_1^c) + F - K$, while the alternative action gives $CS(q)$. The former dominates the latter if $CS(q_1^c) - CS(q) + F - K > 0$, which is again satisfied if $F > K$. Hence consumers' best reply is $\beta_D(q) = 1$ for any $q \neq \{q_2^c, q_1^c\}$. Industry optimal strategy is to play either q_1^c or q_2^c since any other choice will be associated to a negative payoff. In fact, in each state θ_i industry profit for any $q \neq \{q_1^c, q_2^c\}$, is $-(K + F)$. □

Proof of Proposition 4

(a) From (20) $\beta_D^*(q_2^c) = 0$ if and only if the following inequality holds

$$\nu(\theta_1|q_2^c)[CS(q_1^c) - CS(q_2^c) + F] < \nu(\theta_2|q_2^c)K \tag{A.20}$$

and if $\lambda_{q_2^c}^1 = \lambda_{q_2^c}^2 = 1$, we have (in case of pooling at q_2^c then $\nu(\theta_1|q_2^c) = \frac{\gamma_1}{\gamma_1 + \gamma_2} = \gamma_1$) $\gamma_1[CS(q_1^c) - CS(q_2^c) + F] < \gamma_2 K$; solving for K gives (26).

(b) From (21) we know that $\lambda_{q_2^c}^{1*} \in [0, 1]$ if $\beta_D(q_2^c) = \frac{\pi(q_2^c, \theta_1)}{\pi(q_2^c, \theta_1) + F + K}$. But from (20) $\beta_D(q_2^c)$ takes this value only if

$$\nu(\theta_1|q_2^c) = \frac{\nu(\theta_2|q_2^c)K}{CS(q_1^c) - CS(q_2^c) + F} \tag{A.21}$$

so that

$$\frac{\gamma_1 \lambda_{q_2^c}^1}{\gamma_1 \lambda_{q_2^c}^1 + \gamma_2} = \frac{\gamma_2 K}{(\gamma_1 \lambda_{q_2^c}^1 + \gamma_2)(CS(q_1^c) - CS(q_2^c) + F)} \quad (A.22)$$

Solving (A.22) for $\lambda_{q_2^c}^1$ we get (27). □

Proof of Proposition 5

We first compare EW^P and EW^D under semi-separating equilibrium in case of delegation, i.e. the expected welfare shown in (29). If we set $EW^D - EW^P$, we get,

$$\begin{aligned} EW^D - EW^P &= \frac{(W_1(q_1^c) - W_2(q_2^c) + F)[\gamma_1 W_1(q_1^c) + \gamma_2 W_2(q_2^c)]}{W_1(q_1^c) - W_2(q_2^c) + F} + \\ &- \frac{\gamma_2 K [W_1(q_1^c) - W_2(q_2^c)]}{W_1(q_1^c) - W_2(q_2^c) + F} - \gamma_1 (W_1(q_1^c) - K) - \gamma_2 (W_2(q_2^c) - K) \end{aligned} \quad (A.23)$$

since $W_i(\tilde{q}_i) = W_i(q_i^c) - K$. By simplifying we obtain

$$EW^D - EW^P = \frac{K[\gamma_1 (W_1(q_1^c) - W_2(q_2^c)) + F]}{W_1(q_1^c) - W_2(q_2^c) + F} > 0 \quad (A.24)$$

so that $EW^D > EW^P$. In case of pooling equilibrium under delegation, we set

$$\begin{aligned} EW^D - EW^P &= \gamma_1 (W_2(q_2^c) + \pi(q_2^c, \theta_1)) + \gamma_2 W_2(q_2^c) + \\ &- \gamma_1 (W_1(q_1^c) - K) - \gamma_2 (W_2(q_2^c) - K) \end{aligned}$$

so that

$$EW^D - EW^P = \gamma_1 [W_2(q_2^c) + \pi(q_2^c, \theta_1)] - \gamma_1 W_1(q_1^c) + K \quad (A.25)$$

and we know from (26) $K > \left(\frac{\gamma_1}{\gamma_2}\right) (W_1(q_1^c) - W_2(q_2^c) + F)$ (since $CS(q_i^c) = W_i(q_i^c)$). Let us assume, by now, that $K = \left(\frac{\gamma_1}{\gamma_2}\right) (W_1(q_1^c) - W_2(q_2^c) + F)$. Hence by substituting the latter in (A.25) we get $EW^D - EW^P = \gamma_1 W_1(q_1^c)(1 - \gamma_2) - \gamma_1 W_2(q_2^c)(1 - \gamma_2) + \gamma_1 \gamma_2 \pi(q_2^c, \theta_1) + \gamma_1 F$, i.e. $EW^D - EW^P = \gamma_1^2 (W_1(q_1^c) - W_2(q_2^c)) + \gamma_1 \gamma_2 \pi(q_2^c, \theta_1) + \gamma_1 F > 0$. Hence if $K = \left(\frac{\gamma_1}{\gamma_2}\right) (W_1(q_1^c) - W_2(q_2^c) + F)$, then $EW^D > EW^P$. However K is greater than this. But from (A.25) $\partial(EW^D - EW^P)/\partial K > 0$, so that $EW^D > EW^P$ also in case of pooling under delegation. □

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