Vertical Differentiation and Innovation Adoption

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Abstract

The paper investigates a duopoly model with vertical differentiation and both Bertrand and Cournot competition where firms choose between process or product innovation. It is shown that under both competitive regimes three equilibria in innovation adoption may arise: two symmetric equilibria, where firms select the same innovation type, and one asymmetric equilibrium, where the high (low) quality firm chooses a product (process) innovation. The asymmetric equilibrium arises because the high quality firm has a greater incentive to adopt a product innovation than the low quality firm, so that it is the first to introduce it. These equilibria have different impacts on the intensity of competition: the latter is not relaxed when both firms adopt a process innovation. Last, we find that the Cournot competitors tend to favor the introduction of a new product w.r.t. the Bertrand competitors.

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1 Introduction

Managers often face a dilemma: is it better to employ the advances in knowledge and technology to produce a higher quality good or to ensure a higher rate of return by exploiting the benefits of lower unit costs? For example, in the aircraft industry quality is represented by the speed, while a larger size yields lower average costs. Boeing and Airbus made different choices on these two options. Airbus is producing the world's biggest airliner, the A380 (555 seats), Boeing is instead going for speed rather than size with its Sonic Cruiser (250 seats, which flies at 98% of the speed of sound). Championing speed rather than size suggests that Boeing thinks most future growth will come from high quality demand (i.e. fast and frequent point–to–point flights); Airbus, by contrast, still sees a healthy market for a relatively low–cost super–jumbo to connect the world's biggest international airports.[1] The above problem can be classified as the choice between introducing a product or a process innovation. The former consists in the production of new goods, while the latter yields a cost saving benefit in the production of an existing good. This paper tackles this problem and tries to explain what factors might be important in a firm's decision to direct investment (e.g. R&D expenditure) towards the introduction of a product innovation or of a process innovation.

We will show that, under both the competition regimes considered (i.e. Bertrand and Cournot), three types of equilibria concerning the innovation game may arise, two symmetric (both firms introduce either a process innovation or a product innovation) and one asymmetric, where the high quality firm introduces a product innovation and the low quality firm a process innovation. The explanation about the determinant of the prevailing equilibria is based on the different incentives that the two firms have about adopting a product innovation: the high quality firm has higher incentives to introduce a product innovation than the low quality firm under both the competitive regimes considered. Some examples confirm that firms selling goods with different qualities follow different market

[1] More information about the Boeing *vs* Airbus challenge can be found in "Towards the wild blue yonder", *The Economist*, 25*th* April 2002.

strategies: high price car manufactures are usually the first to introduce new optional (e.g. CD players, satellite navigators, ABS, etc.), supermarket chains with a good reputation are the first to adopt quality standards, while hard discounts make of price reductions (through costs savings) their mission.

A model where firms strategically choose between either a process or a product innovation can also supply some additional insights about the effects of that decision on the intensity of competition between the two firms. Under both the competitive regimes considered the three above equilibria in innovation adoption have different impacts on the post–innovation prices. The intensity of competition is not relaxed if both firms adopt a process innovation, i.e. they end up with lower prices than the *status quo* levels. On the contrary, price competition becomes less intense if the high quality firm introduces a product innovation and the low quality firm a costs saving innovation and if both firms adopt a new product. Hence, since the adoption of different types of innovation creates an efficiency gap between the two firms, it follows that costs heterogeneity relaxes price competition, and so it can be classified as a supply side effect: *firms strategically choose to have an efficiency gap rather than costs homogeneity because competition becomes less intense.*[2]

Notwithstanding the relevance of this issue there exist almost no attempts to deal with it, since the literature has usually treated the two kinds of innovation separately. Bonanno and Haworth [1998] (henceforth BH) has provided, up to now, the closest contribution to our work. They find that the *type of competitive regime* in which the firms find themselves (Cournot *vs.* Bertrand) may explain why a firm decides to adopt a product innovation and not a process innovation (and vice versa). They study this problem in a vertically differentiated duopoly where only one firm can innovate. BH show that *if the innovator is the high quality firm* a tendency to favor product innovation emerges in case of Bertrand competition

[2]These results are obtained in a duopoly model with vertical differentiation where the market is uncovered, but they also apply to the covered configuration. The literature (see Choi and Shin [1992], Wauthy [1996] and Ecchia and Lambertini [1998]) has shown that the choice of the market configuration (covered or uncovered) is endogenous. A market is covered if all consumers with a positive willingness to pay for the good buy it, while it is uncovered if some consumers do not purchase the good.

and to favor process innovation in presence of Cournot competition.[3] On the other hand, *if the innovator is the low quality firm* and whenever the two regimes lead to different adoptions, the Bertrand competitor chooses to introduce a process innovation, while the Cournot competitor introduces a product innovation.

Few other contributions have weaker links with our work. Rosenkranz [2003] studies, in a Cournot duopoly model with horizontal differentiation, how two competitors will optimally invest into both process and product innovation.[4] She shows that an increase in consumers' reservation price causes firms to increase R&D investments but also to shift them towards product innovation if the relative efficiency of the two types of innovation is kept constant. Battaggion and Tedeschi [1998][5] investigate a Bertrand duopoly with vertical differentiation and focus on the effects of the different types of innovation on the degree of vertical differentiation. They do not study the strategic choice of two rival firms about the innovation adoption, but show that a symmetric adoption of a product (process) innovation will decrease (increase) the degree of vertical differentiation. Lambertini and Orsini [2000] analyze the incentives to introduce a product innovation or a process innovation in a vertical differentiated monopoly (and so there is no strategic interaction).[6] Weiß [2002] presents a duopoly model with horizontal differentiation where firms choose between a process or product innovation. The latter consists in fixing the profit maximizing variety (while the pre–innovation variety is not optimal). In her framework the competitors are engaged in a

[3]In general if the innovator is the high quality firm one of three things may happen: (1) both the Cournot competitor and the Bertrand competitor choose the process innovation; or (2) both select the product innovation; or (3) they make different choices. Under the latter case the Bertrand (Cournot) competitor choose to introduce a product (process) innovation.

[4]She develops the idea that usually firms have a portfolio of R&D projects, some more targeted at process innovations and some at product innovation, so that the optimal mix between these two types of innovation becomes a key variable in the competitive environment.

[5]Their paper is a contribution to that stream of research (e.g. Athey and Schmutzler [1995] and Eswaran and Gallini [1996]) where a process (product) innovation has the same effect on the quantity supplied by the adopting firm and on the quality of the good of a regressive product (process) innovation, i.e. of a change in technology which reduces the good's quality.

[6]They show that the social planner and the monopolist might adopt different type of innovation.

two–stage competition, i.e. first they select the type of innovation and then they compete in price. She shows that all feasible moves in innovation adoption may belong to the equilibrium path and that the intensity of competition affects the equilibrium selection (if competition is intense (modest) firms choose product (process) innovation, while if it is intermediate they select asymmetrically).

This paper is an attempt to extend BH's results by considering that, in an oligopolistic environment with vertical differentiation, the choice between a product or a process innovation is taken simultaneously by all the firms in an industry. We shall think of process innovation as a reduction in the firm's production costs, so that it can be defined as *costs saving effect* on firm's efficiency. Product innovation will be interpreted as an improvement in the quality of a firm's product (e.g. the introduction of a sonic cruiser aircraft), and we label this as *quality effect*. We will show that BH's results, in a framework where *both* firms innovate, are no longer valid for the high quality firm. Our analysis displays that the latter, regardless the type of innovation chosen by the low quality firm, decides to introduce a product innovation *before* under Cournot than under Bertrand. Hence the high quality firm has a tendency to favor product innovation. Moreover, we find that the Cournot competitors tend to favor the introduction of a new product w.r.t. the Bertrand competitors.

The paper is organized as follows. Section 2 presents the model, Section 3 analyzes the strategic choice between product and process innovation under Bertrand competition. The investigation is divided in two subsections: the pre–innovation equilibrium, where the two firms' qualities are determined (Section 3.1) and the solution of the innovation game (Section 3.2). Section 4 studies the Cournot case, again splitted in the pre–innovation equilibrium (Section 4.1) and in the innovation equilibrium (Section 4.2). Section 5 presents the main results of the paper, while their proofs are reported in the Appendix.

2 The model

Consider a two–stage duopoly model where firms, given a pre–innovation quality pair $\{\theta_H^0, \theta_L^0\}$ with $\theta_H^0 \gg \theta_L^0$ ("0" indicates the *status quo*) sell a vertically differentiated good and may be engaged in Bertrand or in Cournot competition. At $t = 1$ firms simultaneously decide whether to adopt a ProCess innovation (PC), or a ProDuct innovation (PD).[7] Hence at time $t = 1$ firm i ($i = H, L$, where L stands for "low" quality firm and H for "high" quality firm[8]) chooses I_i, where

$$I_i = \begin{cases} 1 & \text{if firm } i \text{ selects } PC \\ 0 & \text{if firm } i \text{ selects } PD \end{cases}$$

This choice affects firm i's costs function if $I_i = 1$ and instead its market share if $I_i = 0$. We consider that quality is a variable cost (Champseaur and Rochet [1989], Gal–Or [1983] and Mussa and Rosen [1978]) so that firms have the following costs function: $C(y_i, \theta_i) = c\frac{\theta_i^2}{2}y_i$, where y_i is the output of firm i, θ_i its quality and c the constant unit costs of production.[9] A process innovation reduces marginal costs (i.e. it has a costs saving effect) by decreasing c; without loss of generality we assume that under $I_i = 1$ production costs become negligible (i.e. $c = 0$). If instead firm i introduces a new product, it benefits from an increase in its quality from $\overline{\theta}_i$ to $\psi\overline{\theta}_i$ with $\psi > 1$ (see BH p. 502). Hence we label ψ as the quality effect. These two effects are exogenous, since we assume that the innovator has invested in R&D (e.g. it has built a lab and hired a team of scientists) and the corresponding costs are sunk.[10] Note that before choosing which type of innovation to adopt firms have the same costs, and that costs homogeneity is maintained if they make the same type of adoption; instead in case of asymmetric adoptions they have different costs functions. Moreover, it follows from our setup that if at $t = 1$ a firm has selected a process innovation its

[7]We rule out the possibility of choosing both types of innovation. Furthermore, the decision not to innovate is not considered since it is always dominated by introducing one of the two innovation types.

[8]The choice of being either the high quality firm or the low quality firm (see Herguera and Lutz [1998]) should be studied in a stage before the choice of innovation. We do not solve this stage, but we assign a label to each firm.

[9]Battaggion and Tedeschi [1998] adopt the same costs function. Our results are valid also for a costs function where quality is a fixed cost (Bonanno [1986], Motta [1993] and Shaked and Sutton [1982, 1983]), e.g. $C(y_i, \theta_i) = cy_i + \frac{\theta_i^2}{2}$, but are based on a simulation analysis. Results are available upon request.

[10]Without loss of generality we assume that R&D costs are equal to 0.

quality remains fixed at the pre–innovation level, i.e. if $I_i = 1 \rightarrow \theta_i = \theta_i^0$ at $t = 2$. At $t = 2$, after observing the rival's innovation choice, under Bertrand (Cournot) firms choose simultaneously the price p_i (quantity y_i).

The market demand is specified as follows: each consumer buys only one unit of the good, and is characterized by the net utility function $U = s\theta - p$, where $s \in [0, 1]$ and p is the price paid for the good. As usual the variable s represents the consumer's willingness to pay (a taste parameter) for the good (Tirole [1988]), and is uniformly distributed over the interval $[0, 1]$. From the above and since the consumer with the lowest willingness to pay is located in 0, he/she will never buy the good, unless $p \leq 0$. Hence the market is always "uncovered" and some consumers are always out of the market. The consumer indifferent between buying the low quality good and not buying at all has a utility given by $s\theta_L - p_L = 0$, so that $s = \frac{p_L}{\theta_L}$. The consumer indifferent between buying the low quality good and the high quality good has a taste parameter equal to $s^* = \frac{p_H - p_L}{\theta_H - \theta_L}$. Hence under Bertrand competition the two firms' market shares are

$$y_H = \left[1 - \frac{p_H - p_L}{\theta_H - \theta_L}\right] \tag{1}$$

$$y_L = \left[\frac{p_H - p_L}{\theta_H - \theta_L} - \frac{p_L}{\theta_L}\right] \tag{2}$$

while under Cournot competition we have

$$p_H = \theta_H(1 - y_H) - \theta_L y_L \tag{3}$$

$$p_L = \theta_L(1 - y_H - y_L) \tag{4}$$

with $y_H + y_L < 1$. Note that in case of product innovation the innovator receives a "market share premium". For instance, in case of price competition, if the innovator is the high quality firm, its new quality is $\psi\theta_H^0$, and so s^* moves towards left since $s^{*'} = \frac{p_H - p_L}{\psi\theta_H^0 - \theta_L^0} < s^*$, i.e. $s^{*'} \rightarrow s$, thereby increasing its market share. If instead the innovator is the low quality firm $s^{*'} = \frac{p_H - p_L}{\theta_H^0 - \psi\theta_L^0} > s^*$, while $s' = \frac{p_L}{\psi\theta_L^0} < s$, i.e. $s' \rightarrow 0$ while $s^{*'} \rightarrow 1$ and so $y_L \uparrow$.

We look for a subgame perfect equilibrium, i.e. a pair of strategies which forms a Nash equilibrium in each subgame. As usual, we compute the solution by backward induction, starting from the last stage of the game, i.e. the Bertrand (or Cournot) subgame. Firm i's profit in the Bertrand subgame is the following (B stands for Bertrand):

$$\begin{aligned}\pi_i^B(I_i, I_j) = \; & I_i I_j \left[p_i y_i(\theta_i^0, \theta_i^0)\right] + I_i(1 - I_j)\left[p_i y_i(\theta_i^0, \psi\theta_j^0)\right] + \\ & +(1 - I_i)I_j \left[\left(p_i - \tfrac{c\theta_i^{0^2}}{2}\right) y_i(\psi\theta_i^0, \theta_i^0)\right] + \\ & +(1 - I_i)(1 - I_j)\left[\left(p_i - \tfrac{c\theta_i^{0^2}}{2}\right) y_i(\psi\theta_i^0, \psi\theta_i^0)\right]\end{aligned} \tag{5}$$

with $i \neq j$ and $i, j = H, L$. Under Cournot competition, the individual profit function is (C is for Cournot):

$$\begin{aligned}\pi_i^C(I_i, I_j) = \; & I_i I_j \left[p_i(\theta_i^0, \theta_i^0) y_i\right] + I_i(1 - I_j)\left[p_i(\theta_i^0, \psi\theta_j^0) y_i\right] + \\ & +(1 - I_i)I_j \left[\left(p_i(\psi\theta_i^0, \theta_i^0) - \tfrac{c\theta_i^{0^2}}{2}\right) y_i\right] + \\ & +(1 - I_i)(1 - I_j)\left[\left(p_i(\psi\theta_i^0, \psi\theta_i^0) - \tfrac{c\theta_i^{0^2}}{2}\right) y_i\right]\end{aligned} \tag{6}$$

again with $i \neq j$ and $i, j = H, L$.

3 Innovation adoption under Bertrand competition

In this Section we investigate the strategic choice between product and process innovation if firms compete in prices in the final market. Since the adoption of a certain type of innovation depends upon the *status quo* quality levels (i.e. θ_H^0, θ_L^0), it is necessary to compute the equilibrium before the innovation game.

3.1 The pre–innovation equilibrium

From (1)–(2) we have

$$\pi_H^B(p_h, p_L, \theta_H, \theta_L) = p_H \left(1 - \frac{p_H - p_L}{\theta_H - \theta_L}\right) - \frac{1}{2} c\theta_H^2 \left(1 - \frac{p_H - p_L}{\theta_H - \theta_L}\right) \tag{7}$$

and

$$\pi_L^B(p_h, p_L, \theta_H, \theta_L) = p_L \left(\frac{p_H - p_L}{\theta_H - \theta_L} - \frac{p_L}{\theta_L}\right) - \frac{1}{2} c\theta_H^2 \left(\frac{p_H - p_L}{\theta_H - \theta_L} - \frac{p_L}{\theta_L}\right) \tag{8}$$

Firms maximize (7)–(8) by choosing first the quality pair (θ_H^*, θ_L^*) and then the market prices (p_H^*, p_L^*). Starting from the bottom stage we have the following FOCs':

$$\frac{\partial \pi_H^B}{\partial p_H} = 1 - \frac{p_H - p_L}{\theta_H - \theta_L} - \frac{p_H}{\theta_H - \theta_L} + \frac{c\theta_H^2}{2(\theta_H - \theta_L)} = 0 \tag{9}$$

$$\frac{\partial \pi_L^B}{\partial p_L} = \frac{p_H - p_L}{\theta_H - \theta_L} - \frac{p_L}{\theta_L} - \left(p_L - \frac{1}{2}c\theta_L^2\right)\left(\frac{1}{\theta_H - \theta_L} + \frac{1}{\theta_L}\right) = 0 \tag{10}$$

Solving the system (9)–(10) gives the pre–innovation equilibrium prices:

$$p_H^* = \frac{\theta_H[4(\theta_H - \theta_L) + c(2\theta_H^2 + \theta_L^2)]}{2(4\theta_H - \theta_L)} \tag{11}$$

$$p_L^* = \frac{\theta_L[2(\theta_H - \theta_L) + c\theta_H(\theta_H + 2\theta_L)]}{2(4\theta_H - \theta_L)} \tag{12}$$

Substituting (11)–(12) in (1)–(2) gives:

$$y_H = \frac{\theta_H[4 - c(2\theta_H + \theta_L)]}{2(4\theta_H - \theta_L)} \tag{13}$$

$$y_L = \frac{\theta_H[2 - c(\theta_H + \theta_L)]}{2(4\theta_H - \theta_L)} \tag{14}$$

Replacing (11)–(12) in (7)–(8) and then differentiating w.r.t θ_H and θ_L yields the following FOCs':

$$\frac{\partial \pi_H^B}{\partial \theta_H} = y_H \frac{4(4\theta_H^2 - 3\theta_H\theta_L + 2\theta_L^2) - c(24\theta_H^3 - 22\theta_H^2\theta_L + 5\theta_H\theta_L^2 + 2\theta_L^3)}{2(4\theta_H - \theta_L)^2} = 0 \tag{15}$$

$$\frac{\partial \pi_L^B}{\partial \theta_L} = y_L \frac{2\theta_H(4\theta_H^2 - 7\theta_L) + c(4\theta_H^3 - 19\theta_H^2\theta_L + 17\theta_H\theta_L^2 - 2\theta_L^3)}{2(4\theta_H - \theta_L)^2} = 0 \tag{16}$$

We can now state the equilibrium qualities in the pre–innovation stage and all the corresponding outcomes for both firms.

Proposition 1 *The Bertrand pre–innovation game has the following equilibrium:* $\theta_H^0 = \frac{0.81952}{c}$, $\theta_L^0 = \frac{0.39872}{c}$, $p_H^0 = \frac{0.45331}{c}$, $p_L^0 = \frac{0.15002}{c}$, $y_H^0 = 0.27924$, $y_L^0 = 0.34450$, $\pi_H^{B0} = \frac{0.03281}{c}$, $\pi_L^{B0} = \frac{0.02430}{c}$.

Proof: See Appendix.

Proposition 1 shows that the high quality firm enjoys a higher profit by selling to the smaller but richer market niche ($y_H^0 < y_L^0$) than the low quality firm; to achieve this higher profit level its quality must be more than double than the quality supplied by the low quality firm to its customers.

3.2 The innovation game

Having solved for the pre–innovation quality levels, we can now compute the subgame perfect equilibrium of the innovation game under price competition. Starting from the last stage of the game, we have to identify the Nash equilibrium in four possible subgames, according to the innovation choices made by the two firms at $t = 1$.

Case a: $I_H = I_L = 1$
Both firms have selected a process innovation at $t = 1$ and so, given that $\theta_H^{11} = \theta_H^0$ and $\theta_L^{11} = \theta_L^0$,[11] the two profit functions, from (5) and from the quality levels stated in Proposition 1, are:

$$\pi_H^B(I_H = I_L = 1) = \pi_H^{B11} = p_H \underbrace{[1 - 2.37643(p_H - p_L)c]}_{y_H} \tag{17}$$

$$\pi_L^B(I_H = I_L = 1) = \pi_L^{B11} = p_L \underbrace{[2.37643(p_H - p_L)c - 2.50803 p_L c]}_{y_L} \tag{18}$$

Hence, by simultaneously solving $\frac{\partial \pi_H^B}{\partial p_H} = 0$ and $\frac{\partial \pi_L^B}{\partial p_L} = 0$, we get the market outcomes shown in Table 1, and, mostly important for our purpose, we identify the two firms' profit functions at $t = 1$ if they both adopt a process innovation, i.e.

$$\pi_H^{B11} = \frac{0.13635}{c} \tag{19}$$

$$\pi_L^{B11} = \frac{0.01658}{c} \tag{20}$$

Case b: $I_H = 0, I_L = 1$

The high quality firm has adopted a product innovation, so that $\theta_H^{01} = \psi\theta_H^0$, while firm L has chosen a process innovation, i.e. $\theta_L^{01} = \theta_L^0$. The two profit functions are:

$$\pi_H^B(I_H = 0, I_L = 1) = \pi_H^{B01} = \left(p_H - \frac{0.33581}{c}\right) \underbrace{\left(1 - \frac{c(p_h - p_L)}{0.81952\psi - 0.39872}\right)}_{y_H}$$

[11] The superscripts indicate the innovation moves at $t = 1$; 11 means that $I_H = 1$ and $I_L = 1$.

Case a: $I_H = I_L = 1$
$p_H^{11} = \frac{0.23953}{c}$
$p_L^{11} = \frac{0.05827}{c}$
$y_H^{11} = 0.56924$
$y_L^{11} = 0.28462$

Case b: $I_H = 0, I_L = 1$
$p_H^{11} = 0.256(10^{-7})\frac{114+0.32115(10^{13})\psi)(0.16006(10^{8})\psi-0.12288(10^{7}))}{c(-0.39062(10^{12})+0.32115(10^{13})\psi)}$
$p_L^{01} = 20000\frac{0.16006(10^{8})\psi-0.12288(10^{7})}{c(-0.39062(10^{12})+0.32115(10^{13})\psi)}$
$y_H^{01} = 0.5\frac{(\psi-0.13010)(\psi-0.76619)}{(\psi-0.12163)(\psi-0.48653)}$
$y_L^{01} = 0.25\frac{\psi(\psi-0.07677)}{(\psi-0.12163)(\psi-0.48653)}$

Case c: $I_H = 1, I_L = 0$
$p_H^{10} = 0.1024(10^{-6})\frac{0.62524(10^{19})\psi-0.13474(10^{20})+0.22194(10^{9})\psi^2}{c(0.39062(10^{12})\psi-0.32115(10^{13}))}$
$p_L^{10} = 0.1024(10^{-6})\frac{-0.31262(10^{19})\psi-0.12465(10^{19})+0.1521(10^{19})\psi^2}{c(0.39062(10^{12})\psi-0.32115(10^{13}))}$
$y_H^{10} = -0.64205(10^{-11})\frac{0.64205(10^{12})\psi-0.13798(10^{13})}{(\psi-2.05538)(\psi-8.22151)}$
$y_L^{10} = -2.05538\frac{(\psi-0.45548)(\psi-1.79926)}{(\psi-2.05538)(\psi-8.22151)}$

Case d: $I_H = I_L = 0$
$p_H^{00} = 0.4904(10^{-19})\frac{0.48844(10^{19})\psi+0.43592(10^{19})}{c}$
$p_L^{00} = 0.5827(10^{-11})\frac{0.1(10^{11})+0.15745(10^{11})}{c}$
$y_H^{00} = \frac{0.56924\psi-0.28999}{\psi}$
$y_L^{00} = \frac{0.28462\psi+0.05988}{\psi}$

Table 1: Market outcomes in Bertrand subgames

$$\pi_L^B(I_H=0,I_L=1)=\pi_L^{B01}=p_L\underbrace{\left(\frac{c(p_h-p_L)}{0.81952\psi-0.39872}-2.50803cp_L\right)}_{y_L}$$

By solving the two FOCs' at $t=2$ we get the market prices and shares reported in Table 1,[12] and the following profit functions at $t=1$:

$$\pi_H^{B01}=0.17317(10^{25})\frac{(\psi-0.1301)^2(\psi-0.76619)^2}{c(0.11254(10^{25})\psi-0.61685(10^{25})\psi^2+0.84524(10^{25})\psi^3-0.60840(10^{23}))} \tag{21}$$

$$\pi_L^{B01}=0.21063(10^{24})\frac{\psi(\psi^2-0.15354\psi+0.00589)}{c(0.11254(10^{25})\psi-0.61685(10^{25})\psi^2+0.84524(10^{25})\psi^3-0.60840(10^{23}))} \tag{22}$$

Case c: $I_H=1,I_L=0$

The two qualities are $\theta_H^{10}=\theta_H^0$ and $\theta_L^{10}=\psi\theta_L^0$. Hence:

$$y_H^{10}=1-\frac{c(p_H-p_L)}{0.81952-0.39872\psi},y_L^{10}=\frac{c(p_H-p_L)}{0.81952-0.39872\psi}-\frac{2.50803}{\psi}c$$

By inspection, since $0<y_i^{10}<1$ then $\psi\neq 2.05538$. The two profit functions are:

$$\pi_H^B(I_H=1,I_L=0)=\pi_H^{B10}=p_H\underbrace{\frac{c(p_H-p_L)}{0.81952-0.39872\psi}}_{y_H}$$

$$\pi_L^B(I_H=1,I_L=0)=\pi_L^{B10}=\left(p_L-\frac{0.07949}{c}\right)\underbrace{\left(\frac{c(p_H-p_L)}{0.81952-0.39872\psi}-\frac{2.50803}{\psi}c\right)}_{y_L}$$

Solving $\frac{\partial\pi_H^{B10}}{\partial p_H}=0$ and $\frac{\partial\pi_L^{B10}}{\partial p_L}=0$ yields the market outcomes shown in Table 1. Note that, since by assumption $y_H\gg 0$, $y_H\ll 1$, $y_L\gg 0$, $y_L\ll 1$ and $y_H+y_L\ll 1$, there exist some restrictions on the ψ–values to have feasible

[12] Note that, from Table 1, $0<y_H^{01}<1$, $0<y_L^{01}<1$ and $y_H^{01}+y_L^{01}<1$ for $\psi>1$, while $p_H^{01}>0$ and $p_L^{01}>0$ for $\psi>1$.

solutions under *Case c*.[13] The latter are $1 \leq \psi \leq 1.7993$ and $2.155 \leq \psi \leq 2.3972$. Last, the two firms' profits at $t = 1$ are:

$$\pi_H^{B10} = -0.14551(10^{14}) \frac{(\psi+0.28171(10^{11})(\psi-2.15501)(\psi-2.15509)}{c(-0.11254(10^{25})\psi^2+0.61685(10^{25})\psi-0.84524(10^{25})+0.6084(10^{23})\psi^3)} \quad (23)$$

$$\pi_L^{B10} = -0.49859(10^{23}) \frac{(\psi-0.45546)(\psi-0.45549)(\psi-1.79919)(\psi-1.79933)}{c\psi(-0.11254(10^{25})\psi^2+0.61685(10^{25})\psi-0.84524(10^{25})+0.6084(10^{23})\psi^3)} \quad (24)$$

Case d: $I_H = I_L = 0$

We have that $\theta_H^{00} = \psi\theta_H^0$ and $\theta_L^{00} = \psi\theta_L^0$. Hence the two profit functions are:

$$\pi_H^B(I_H = I_L = 0) = \pi_H^{B00} = \left(p_H - \frac{0.33581}{c}\right) \underbrace{\left(1 - \frac{2.37643(p_H - p_L)c}{\psi}\right)}_{y_H}$$

$$\pi_L^B(I_H = I_L = 0) = \pi_L^{B00} = \left(p_L - \frac{0.07949}{c}\right) \underbrace{\left(\frac{2.37643(p_H - p_L)c}{\psi} - \frac{2.50803p_Lc}{\psi}\right)}_{y_H}$$

Solving the game at $t = 2$ gives the market outcomes displayed in Table 1 and the following profits at $t = 1$:

$$\pi_H^{B00} = 0.13635 \frac{(\psi-0.50942)(\psi-0.50945)}{c\psi} \quad (25)$$

$$\pi_L^{B00} = 0.01658 \frac{(\psi+0.2104)(\psi+0.2103)}{c\psi} \quad (26)$$

Having identified the reduced form of firm i's profit under each possible innovation moves at $t = 1$, we can now identify the subgame perfect equilibrium. First we have to compute firm H's best reply to $I_L = 1$ and to $I_L = 0$. If firm L selects $I_L = 1$, then by comparing (19) and (21) we get that $\pi_H^{B11} \geq \pi_H^{B01}$ if $1 \leq \psi \leq 1.65168$. Hence

$$I_H^B(I_L = 1) = \begin{cases} 1 & \text{if} \quad 1 \leq \psi \leq 1.65168 \\ 0 & \text{otherwise} \end{cases} \quad (27)$$

[13] Computation yields that $y_H^{10} \leq 1$ if $1 \leq \psi \leq 1.8723$ and if $2.05538 < \psi \leq 4.2938$, that $y_H^{10} \geq 0$ if $1 \leq \psi < 2.05538$ and if $2.155 \leq \psi < 8.22151$. Moreover, $y_L^{10} \leq 1$ if $1 \leq \psi < 2.05538$, if $2.1485 \leq \psi \leq 6.1994$ and if $\psi > 8.22151$, while $y_L^{10} \geq 0$ if $1 \leq \psi \leq 1.7993$ and if $2.05538 < \psi < 8.22151$. Last, $y_H^{10} + y_L^{10} \leq 1$ if $1 \leq \psi \leq 2.3972$ and if $\psi > 8.22151$.

If instead firm L chooses $I_L = 0$, we have, from (23) and (25), that $\pi_H^{B10} \geq \pi_H^{B00}$ if $1 \leq \psi \leq 1.61099$. Note that from the analysis developed under Case c, to have a feasible solution if $\{I_H = 1, I_L = 0\}$, we require that $1 \leq \psi \leq 1.7993$ and $2.155 \leq \psi \leq 2.3972$; however under the latter interval we get that $\pi_H^{B10} \ll 0$. Therefore:

$$I_H^B(I_L = 0) = \begin{cases} 1 & \text{if} \quad 1 \leq \psi \leq 1.61099 \\ 0 & \text{otherwise} \end{cases} \tag{28}$$

Last, we need to compute firm L's best reply to $I_H = 1$ and to $I_H = 0$. If firm H adopts a process innovation, solving $\pi_L^{B11} \geq \pi_L^{B10}$ yields (from (20) and (24)) that for firm L choosing a process innovation always dominates the alternative. Under $\{I_H = 1, I_L = 0\}$ a feasible solution exists only if $1 \leq \psi \leq 1.7993$ and $2.155 \leq \psi \leq 2.3972$; however under the former interval $\pi_L^{B11} \gg \pi_L^{B10}$, while in the latter interval $\pi_L^{B10} \ll 0$. Hence $I_L^B(I_H = 1) = 1$. If instead firm H selects a product innovation, by comparing (22) and (26) we have that $\pi_L^{B01} \geq \pi_L^{B00}$ when $\psi \leq 1.75087$, i.e.

$$I_L^B(I_H = 0) = \begin{cases} 1 & \text{if} \quad 1 \leq \psi \leq 1.75087 \\ 0 & \text{otherwise} \end{cases} \tag{29}$$

Now we can identify the equilibrium in innovation adoption.

Proposition 2 *In case of Bertrand competition, the innovation game has the following equilibria:*
i. $(I_H^* = I_L^* = 1)$ *if* $1 \leq \psi \leq 1.65168$*;*
ii. $(I_H^* = 0, I_L^* = 1)$ *if* $1.65168 < \psi \leq 1.75087$*;*
iii. $(I_H^* = I_L^* = 0)$ *if* $\psi > 1.75087$.

Proof: See Appendix.

Proposition 2 states that only three equilibria can arise in the innovation game under Bertrand competition: two symmetric equilibria (where both firms adopt either a process innovation or a product innovation) and only one asymmetric equilibrium (where the high quality firm adopts a product innovation and the low quality firm a process innovation). Moreover it highlights that the high quality firm is the first to adopt a product innovation, and that there exists an interval where the low quality firm still finds profitable to benefit from a unit costs reduction and not from a product innovation. Note that when the equilibrium is $(I_H^* = 0, I_L^* = 1)$ the two competitors have costs heterogeneity. By comparing the *status quo* (i.e. the pre–innovation equilibrium) and the market outcomes under each innovation equilibrium we can draw several interesting implications.

First, under the symmetric equilibrium $I_H^* = I_L^* = 1$ we have that: $p_H^{11} = \frac{0.23953}{c} < p_H^0$, $p_L^{11} = \frac{0.05827}{c} < p_L^0$, $y_H^{11} = 0.56924 > y_H^0$, $y_L^{11} = 0.28462 < y_L^0$, $\pi_H^{B11} = \frac{0.13635}{c} > \pi_H^{B0}$ and $\pi_L^{B11} = \frac{0.01658}{c} < \pi_L^{B0}$. Hence market prices decrease leading to an increase in the intensity of competition between the two firms. For both firms the cost savings effect has a negative strategic effect: the competitor responds to a reduction in firm i's costs by reducing its own price, thereby increasing the intensity of competition. The high quality firm benefits from this: its market share rises up as well as its profits. On the contrary, the low quality firm suffers of a profit loss compared with the *status quo*: since both goods have the same quality than in the *status quo* but are sold at lower prices, more consumers buy the high quality good.[14] The price reduction operated by the low quality firm is not enough to attract more consumers towards its good.

Second, under the asymmetric equilibrium $\{I_H^* = 0, I_L^* = 1\}$, it is straightforward to show, by plotting p_H^{01} and p_L^{01} (see Table 1) in the interval $1.65168 \le \psi \le 1.75087$, that p_H^{01} increases with ψ, while p_L^{01} shrinks. Moreover, along the same interval, $p_H^{01} \gg p_H^0$ while $p_L^{01} \ll p_L^0$. Hence price competition is softer than in the *status quo*. Meanwhile, y_H^{01} (y_L^{01}) increases (decreases) with ψ, and both market share are higher than the corresponding levels in the pre–innovation stage. Last $\pi_H^{B01} > \pi_H^{B0}$ and $\pi_L^{B01} > \pi_L^{B0}$ (with $\pi_H^{B01} > \pi_L^{B01}$). Hence both firms benefit from the reduction in competition due to asymmetric adoption and costs heterogeneity.

[14]Since firm H adopts a process innovation under this equilibrium, firm L gets a reduction in its profits compared with the pre–innovation level. However, its profits would be lower if it chooses to remain at the *status quo*: in this case firm L would have the same pre–innovation quality and no cost savings, suffering from an efficiency gap from the high quality firm. If $I_H = 1$ and firm L stays fixed at the *status quo*, its costs are $c\frac{\theta_H^0}{2}y_L$ and the solution at $t = 2$ is $y_H = 0.623, y_L = 0.11737$, and, above all, $\pi_L = \frac{0.00282}{c} < \pi_L^{B11}$. Hence adopting an innovation is always a *dominating* strategy.

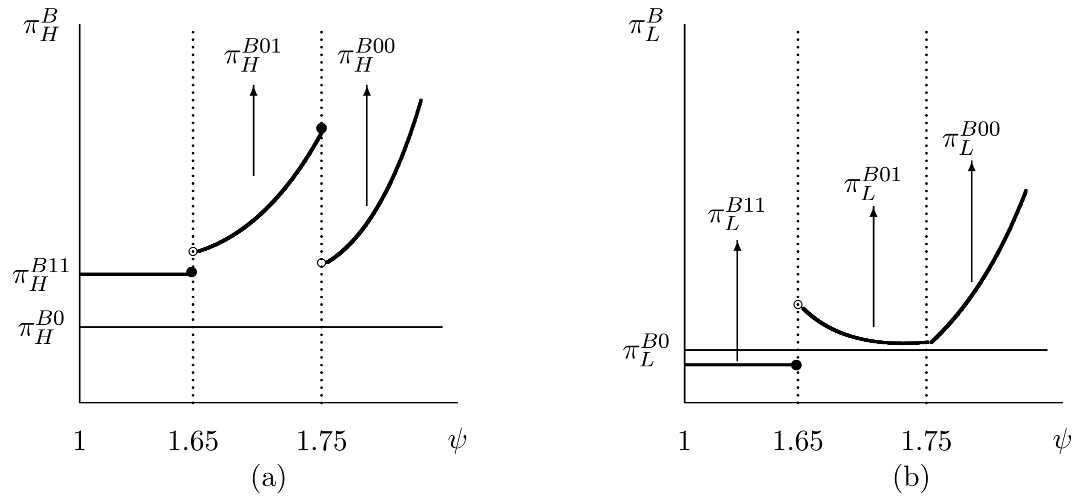

Figure 1: Firms' profits under the innovation equilibria—Bertrand

Last, if both firms adopt a product innovation, we get that in the interval $\psi > 1.75087$ both market prices arise with ψ and are higher than the *status quo*, as well as the two firms' market shares. Moreover, this market share premium is profitable for both firms (i.e. $\pi_i^{B00} > \pi_i^{B0}$). Figure 1 shows the profitability of the three equilibria for the two firms.

4 Innovation adoption under Cournot competition

In this Section we analyze the innovation game when firms compete *á la* Cournot in the final market. Before solving it we need to compute the two pre–innovation quality levels.

4.1 The pre–innovation equilibrium

Under quantity competition the two firms' market demand are (3)–(4), and so the two firms' profits are:

$$\pi_H^C(y_H, y_L, \theta_H, \theta_L) = (\theta_H - \theta_L y_H - \theta_L y_L)y_H - \frac{1}{2}c\theta_H^2 y_H \tag{30}$$

$$\pi_L^C(y_H, y_L, \theta_H, \theta_L) = (\theta_L - \theta_L y_H - \theta_L y_L)y_L - \frac{1}{2}c\theta_L^2 y_L \tag{31}$$

Firms maximize (30)–(31) w.r.t. (θ_H, θ_L) first and then w.r.t. (y_H, y_L). By solving the Cournot subgame we have that:

$$\frac{\partial \pi_H^C}{\partial y_H} = \theta_H - 2\theta_H y_H - \theta_L y_L - \frac{1}{2} c\theta_H^2 = 0 \tag{32}$$

$$\frac{\partial \pi_L^C}{\partial y_L} = \theta_L - 2\theta_L y_L - \theta_L y_H - \frac{1}{2} c\theta_L^2 = 0 \tag{33}$$

Solving the system (32)–(33) gives the pre–innovation market shares:

$$y_H^* = \frac{2(2\theta_H - \theta_L) - c(2\theta_H^2 - \theta_L^2)}{2(4\theta_H - \theta_L)} \tag{34}$$

$$y_L^* = \frac{\theta_H(2 + c(\theta_H - 2\theta_L))}{2(4\theta_H - \theta_L)} \tag{35}$$

After substituting y_H^*, y_L^* in (30)–(31) and then differentiating w.r.t. θ_H, θ_L we get

$$\frac{\partial \pi_H^C}{\partial \theta_H} = y_H \frac{16\theta_H^2 - 4\theta_H\theta_L + 2\theta_L^2 - c(24\theta_H^3 - 10\theta_H^2\theta_L + 4\theta_H\theta_L^2 + \theta_L^3)}{2(4\theta_H - \theta_L)^2} = 0 \tag{36}$$

$$\frac{\partial \pi_L^C}{\partial \theta_L} = y_L \frac{\theta_H(8\theta_H - 2\theta_L + c(4\theta_H^2 - 23\theta_H\theta_L + 2\theta_L^2))}{2(4\theta_H - \theta_L)^2} = 0 \tag{37}$$

The following Proposition points out the equilibrium in the pre–innovation stage.

Proposition 3 *The Cournot pre–innovation game has the following equilibrium:* $\theta_H^0 = \frac{0.73810}{c}$, $\theta_L^0 = \frac{0.58558}{c}$, $p_H^0 = \frac{0.43371}{c}$, $p_L^0 = \frac{0.31452}{c}$, $y_H^0 = 0.21856$, $y_L^0 = 0.24433$, $\pi_H^{C0} = \frac{0.03526}{c}$, $\pi_L^{C0} = \frac{0.03496}{c}$.

Proof: See Appendix.

Again the high quality firm has a lower but more profitable market share than the low quality firm, due to a quality level about 25% greater than its rival. Moreover, both firms enjoy higher profits than under Bertrand.

4.2 The innovation game

The two firms have to decide simultaneously which type of innovation to adopt when they compete *à la* Cournot and have their qualities set at θ_H^0, θ_L^0. To identify the solutions we apply the same procedure shown in Section 3.2; hence a less detailed explanation is provided here. Again there are four possible Cournot subgames, according to the innovation choices made at $t = 1$. In each subgame the firms' profit functions vary according to (6), after substituting for each possible $\{I_H, I_L\}$ pair.

Case a: $I_H = I_L = 1$

In this subgame the two profit functions are:

$$\pi_H^{B11} = \left(\frac{0.73810}{c} - \frac{0.73810}{c}y_H - \frac{0.58558}{c}y_L\right) y_H$$

$$\pi_L^{C11} = \left(\frac{0.58558}{c} - \frac{0.58558}{c}y_H - \frac{0.58558}{c}y_L\right) y_L$$

and, by solving the FOCs' $\frac{\partial \pi_H^C}{\partial y_H} = 0, \frac{\partial \pi_L^C}{\partial y_L} = 0$, we obtain the market outcomes shown in Table 2, and the following profits at $t = 1$

$$\pi_H^{C11} = \frac{0.10451}{c} \tag{38}$$

$$\pi_L^{C11} = \frac{0.05695}{c} \tag{39}$$

Case b: $I_H = 0, I_L = 1$

At $t = 2$ we have:

$$\pi_H^{C01} = \left(\frac{0.73810\psi}{c} - \frac{0.73810\psi}{c}y_H - \frac{0.58558}{c}y_L\right) y_H - \frac{0.27240}{c}y_H$$

$$\pi_L^{C01} = \left(\frac{0.58558}{c} - \frac{0.58558}{c}y_H - \frac{0.58558}{c}y_L\right) y_L$$

The market outcomes at $t = 2$ are reported in Table 2, while the corresponding profits at $t = 1$ are:

$$\pi_H^{C01} = 0.40211(10^{10})\frac{(\psi + 0.47943(10^{-10}))(\psi - 0.76572)(\psi - 0.76574)}{c(0.21792(10^{11})\psi^2 - 0.86443(10^{10})\psi + 0.85726(10^9))} \tag{40}$$

Case a: $I_H = I_L = 1$
$p_H^{11} = \frac{0.27774}{c}$
$p_L^{11} = \frac{0.18261}{c}$
$y_H^{11} = 0.37629$
$y_L^{11} = 0.31185$

Case b: $I_H = 0, I_L = 1$
$p_H^{11} = 54479.16\frac{(\psi+0.36905)(\psi-0.39668)}{c(147620\psi-29279)}$
$p_L^{01} = \frac{21610.83\psi+7975.48}{c(147620\psi-29279)}$
$y_H^{01} = 0.0005\frac{0.14762(10^9)\psi-0.11304(10^9)}{147620\psi-29279}$
$y_L^{01} = 1.84525\frac{20000\psi+7381}{147620\psi-29279}$

Case c: $I_H = 1, I_L = 0$
$p_H^{10} = \frac{-60806.60+21610.83\psi}{c(-147620+29279\psi)}$
$p_L^{10} = -\frac{16590.89\psi+12654.87}{c(-147620+29279\psi)}$
$y_H^{10} = 0.00001\frac{0.29279(10^{10})\psi-0.82383(10^{10})}{-147620+29279\psi}$
$y_L^{10} = -0.73810\frac{50000\psi-29279}{\psi(-147620+29279\psi)}$

Case d: $I_H = I_L = 0$
$p_H^{00} = \frac{0.27774\psi+0.15597}{c}$
$p_L^{00} = \frac{0.18261\psi+0.13191}{c}$
$y_H^{00} = 0.84501(10^{-10})\frac{-0.18667(10^{10})+0.44531(10^{10})\psi}{\psi}$
$y_L^{00} = 0.31185(10^{-5})\frac{-21653+100000\psi}{\psi}$

Table 2: Market outcomes in Cournot subgames

$$\pi_L^{C01} = 0.79755(10^9)\frac{(\psi + 0.36906)(\psi + 0.36904)}{c(0.21792(10^{11})\psi^2 - 0.86443(10^{10})\psi + 0.85726(10^9))} \tag{41}$$

Case c: $I_H = 1, I_L = 0$

The profit functions at $t = 2$ are (from (6)):

$$\pi_H^{C10} = \left(\frac{0.73810}{c} - \frac{0.73810\psi}{c}y_H - \frac{0.58558\psi}{c}y_L\right)y_H$$

$$\pi_L^{C10} = \left(\frac{0.58558\psi}{c} - \frac{0.58558\psi}{c}y_H - \frac{0.58558\psi}{c}y_L\right)y_L - \frac{0.17145}{c}y_L$$

The subgame solutions at $t = 2$ are displayed in Table 2. Again, since by assumption $y_H \gg 0$, $y_H \ll 1$, $y_L \gg 0$, $y_L \ll 1$ and $y_H + y_L \ll 1$, some restrictions on the ψ–values have to be imposed to achieve feasible solutions under *Case c*.[15] Under Cournot competition the feasible ψ–values in this subgame are those falling in the interval $\psi \in [1, 2.8137]$. Firms' profits at $t = 1$ are:

$$\pi_H^{C10} = 0.63274(10^9)\frac{(\psi - 2.81367)^2)}{c(0.21792(10^{11}) - 0.86443(10^{10})\psi + 0.85726(10^9)\psi^2)} \tag{42}$$

$$\pi_L^{C10} = \frac{0.79756(10^9)\psi^2 - 0.93406(10^9)\psi + 0.27348(10^9)}{c\psi(0.21792(10^{11}) - 0.86443(10^{10})\psi + 0.85726(10^9)\psi^2)} \tag{43}$$

Case d: $I_H = I_L = 0$

If both firms decide to introduce a product innovation, the profit functions are:

$$\pi_H^{C00} = \left(\frac{0.73810\psi}{c} - \frac{0.73810\psi}{c}y_H - \frac{0.58558\psi}{c}y_L\right)y_H - \frac{0.27240}{c}y_H$$

$$\pi_L^{C00} = \left(\frac{0.58558\psi}{c} - \frac{0.58558\psi}{c}y_H - \frac{0.58558\psi}{c}y_L\right)y_L - \frac{0.17145}{c}y_L$$

[15]First of all we need that $\psi \neq 5.0418$ to avoid asymptotic solutions at y_i, p_i. Second, computation yields that $y_H^{10} \leq 1$ if $1 \leq \psi \leq 5.0418$, that $y_H^{10} \geq 0$ if $1 \leq \psi < 2.8137$ and if $\psi > 5.0418$. Moreover, $y_L^{10} \leq 1$ if $1 \leq \psi < 3.9674$, and if $\psi > 5.0418$, while $y_L^{10} \geq 0$ if $1 \leq \psi < 5.0418$. Last, $y_H^{10} + y_L^{10} \leq 1$ if $1 \leq \psi < 5.0418$.

The market outcomes in the final stage are shown in Table 2, while the two profits are:

$$\pi_H^{C00} = \frac{0.10451\psi^2 - 0.08762\psi + 0.01836)}{c\psi} \tag{44}$$

$$\pi_L^{C00} = 0.05695\frac{(\psi - 0.21652)(\psi - 0.21654)}{c\psi} \tag{45}$$

Now we compute each firm's best reply. We get $I_H(I_L = 1)$ by comparing (38) and (40), so that

$$I_H^C(I_L = 1) = \begin{cases} 1 & \text{if} \quad 1 \leq \psi \leq 1.5995 \\ 0 & \text{otherwise} \end{cases} \tag{46}$$

while (from (42) and (44)

$$I_H^C(I_L = 0) = \begin{cases} 1 & \text{if} \quad 1 \leq \psi \leq 1.6036 \\ 0 & \text{otherwise} \end{cases} \tag{47}$$

Moreover, from (39) and (43) we have that

$$I_L^C(I_H = 1) = \begin{cases} 1 & \text{if} \quad 1 \leq \psi \leq 1.6636 \quad \text{and if} \quad \psi > 2.8137 \\ 0 & \text{if} \quad 1.6636 < \psi \leq 2.8137 \end{cases} \tag{48}$$

while by comparing (41) and (45) we obtain that

$$I_L^C(I_H = 0) = \begin{cases} 1 & \text{if} \quad 1 \leq \psi \leq 1.6485 \\ 0 & \text{otherwise} \end{cases} \tag{49}$$

The equilibrium of the innovation game under Cournot equilibrium is the following one:

Proposition 4 *In case of Cournot competition, the innovation game has the following equilibria:*
i. $(I_H^* = I_L^* = 1)$ *if* $1 \leq \psi \leq 1.5995$*;*
ii. $(I_H^* = 0, I_L^* = 1)$ *if* $1.5995 < \psi \leq 1.6485$*;*
iii. $(I_H^* = I_L^* = 0)$ *if* $\psi > 1.6485$.

Proof: See Appendix.

Proposition 4 points out that also in case of Cournot competition three equilibria arise in the innovation game, similarly to the Bertrand regime. However, by comparing Proposition 2 and Proposition 4, the interval where both firms adopt a process innovation, and the interval where the high quality firm introduces a new product while its rival chooses a cost saving innovation, are smaller under Cournot than under Bertrand. Consequently, the interval where both firms decide to introduce a new product is larger under Cournot than under Bertrand, i.e. both firms select $I_i^* = 0$ *before* in case of quantity competition than in case of price competition. Hence, we add a new insight to the BH's results: the Cournot competitors tend to favor a product innovation in comparison with the Bertrand competitors, *when they both adopt an innovation*.

Moreover, if we compare the market outcomes after the innovation adoptions with the *status quo*, and if both firms choose a cost saving innovation, we get that market prices are lower than the pre–innovation levels, while both market shares are greater. Furthermore, both firms benefit of a profit increase. This is a relevant difference between the Cournot and the Bertrand case: under the latter, if $I_H^* = I_L^* = 1$, the low quality firm has a profit reduction w.r.t. the *status quo*. Under Cournot the introduction of a process innovation increases the intensity of competition but this effect is outperformed by the increase in the market shares and by the reduction of production costs.

If the innovation equilibrium is asymmetric, we have the same effects than under Bertrand, with the unique difference that the low quality firm market shares decreases with ψ, but it is always greater than the pre–innovation level. Hence Figure 1 can be applied also to the Cournot regime, with the unique exception that in case of symmetric adoption of a process innovation *also* the low quality firm enjoys a profit increase w.r.t. the *status quo*. Last, profits are always greater under Cournot than under Bertrand.

Proposition 2 and Proposition 4 highlight the interesting result that firms, independently of the competitive regime, might choose asymmetrically between product and process innovation. Hence this contribution points out that firms selling goods with a quality gap have different incentives in adopting a product innovation, since introducing the latter becomes a dominant strategy for the high quality firm before than for the quality follower. It is crucial to shed light some intuitions on this issue. BH, for instance, have provided an answer based upon the competitive regime, but in model where there is only one innovator. We have instead obtained that, when both firms innovate, the competitive regime is not a factor explaining the firms' different attitudes toward the two types of innovation. Our explanation is based on two factors: (1) the impact of each type of innovation on the firm's market share and (2) on its unit profit margin. The impact of each innovation type on these factors is different for the two firms.

To understand why, consider first the high quality firm. We know that the symmetric adoption of a process innovation generate a reduction of both prices and a rather strong increase in the high quality firm market share; moreover, the latter enjoys an increase in its unit margin because the price reduction is more than offset by the decrease in production costs. On the contrary, if the low quality firm chooses a process innovation and the high quality firm a product innovation, the two market prices go in opposite directions (i.e. p_H rises while p_L shrinks), firm H's market share increases, while its unit margin widens. However, if the quality effect is small, the increase in firm H's market share and unit margin obtained by the introduction of a new product are smaller than those granted by a process innovation; if ψ is small also the high quality firm chooses a costs reduction. As the quality effect rises the unit margin enlarges more than proportionally (while it remains fixed if firm H selects a process innovation) and so there is a critical level of ψ such that the introduction of a new product becomes more profitable for the high quality firm.

The picture is rather different if we consider the low quality firm. If both firms choose a process innovation firm L suffers of a reduction both in the market share and in the unit margin. The latter is due to a price decrease higher than the costs reduction. However the landscape becomes worse if the low quality firm introduces a new good when the high quality firm has selected a process innovation:[16] in this case both prices shrink but consumers reward the high quality

[16] We do not consider the possibility of leapfrogging, as in BH. For a discussion see the Remark

firm (a higher quality good is sold at lower price), so that firm L's market share drops dramatically. In addition, also the unit margin falls. Hence the low quality firm has never the incentive to be the first to introduce a new product. Something different happens if the high quality firm has chosen a product innovation. In this case if firm L selects a process innovation its market share falls but its unit margin increases (because its price reduction is lower than the unit costs reduction). Instead if firm L adopts a product innovation its market share rises but the increase in the unit margin is lower than that obtained with a process innovation. Hence, for a specific interval of the quality effect, firm L considers more profitable the process innovation because it grants a unit margin increase much bigger than the combination of the two above factors (i.e. greater market share plus higher margin) in case of product innovation. If instead the quality effect increases these two effects, sooner or later will overcome the profitability of the costs reduction also for the low quality firm. As a result, both firms end up with introducing a new product. These observations points out that the high quality firm is really the market leader since it is always the first to improve the quality embedded in the vertical differentiated market.

Remark

The above insights might change if we consider the possibility of leapfrogging (i.e. a change in the identity of the high quality firm).[17] We do not tackle this issue since we assign labels to each firms: firm H is the *high* quality firm, while firm L is the *low* quality firm, i.e. we do not consider the problem of the "identity" of the high quality firm. Our attention focuses on which type of innovation adopts the *high* quality firm and, simultaneously, the *low* quality firm,[18] i.e. it does not

below.

[17]As shown by Choi and Shin [1992], a problem arising in duopolistic models of vertical product differentiation and identical firms is that there exist two symmetric market equilibria in pure strategy (and one in mixed strategy). In one equilibrium firm i is the high quality firm while in the other equilibrium is the low quality firm. In order to ensure a unique equilibrium the investigation is either restricted to marginal analysis in the vicinity of one of these equilibria via technological constraints or quality' leadership is assigned at the beginning of the game and taken as given.

[18]For example, in the symmetric equilibrium ($I_H^* = I_L^* = 1$) the quality leader and follower

matter *who* is the high quality firm, but what type of innovation the leader (and the follower) adopts.[19]

5 Conclusions

This paper investigates a duopoly model of vertical differentiation where firms simultaneously select whether to adopt a process innovation or a product innovation. This decision is taken both in case of Bertrand and Cournot competition. The two innovations have different impacts on firm's profitability, identified by a costs saving effect (process innovation) and by a market share premium (product innovation). The analysis has produced the following results: First, under both competitive regimes three equilibria in the innovation game may arise: two symmetric (where both firms choose either a process or a product innovation) and one asymmetric (where the high (low) quality firm selects a product (process) innovation. Second, asymmetric equilibria in innovation adoption arise because the high quality firm is the first to adopt a product innovation, regardless the type of competitive regime. The two factors explaining these different behaviors are (1) the impact of each innovation on firms' market shares and (2) on their unit margins. Third, all equilibria yield, in general, a profits increase w.r.t. to the pre–innovation levels. Fourth, in contrast with Bonanno and Haworth [1998] which consider only one innovator, both firms display a tendency to favor product innovation under Cournot. Last, the above equilibria have different effects on the intensity of competition: specifically, the latter is not relaxed only in case of a symmetric adoption of a costs saving innovation, while it is always soften when firms choose asymmetrically. Since under this equilibrium firms endogenize an efficiency gap, costs heterogeneity may be regarded as a supply side effect

choose the same type of innovation.

[19]The literature has also pointed out that leapfrogging might arise in case of a change of some parameters (e.g. a reduction in trade barriers as in Motta, Cabrales and Thisse [1997]). However, in such a case a variation in the quality provided and in the identity of the high quality firm usually involves an adjustment costs. Hence in our settings we should introduce a fixed costs of changing the identity, and it will always be possible to identify a level sufficiently high of this costs that no leapfrogging occurs.

implemented to shrink price competition.

6 Appendix

Proof of Proposition 1: Since $y_H \gg 0$ and $y_L \gg 0$ and in both FOCs' the denominator is positive, (15)–(16) are simultaneously satisfied when

$$4(4\theta_H^2 - 3\theta_H\theta_L + 2\theta_L^2) - c(24\theta_H^3 - 22\theta_H^2\theta_L + 5\theta_H\theta_L^2 + 2\theta_L^3) = 0$$

and

$$2\theta_H(4\theta_H^2 - 7\theta_L) + c(4\theta_H^3 - 19\theta_H^2\theta_L + 17\theta_H\theta_L^2 - 2\theta_L^3) = 0$$

Solving this system yields three solutions: $\{\theta_H = \frac{2}{3}\frac{1}{c}, \theta_L = \frac{8}{3}\frac{1}{c}\}$, $\{\theta_H = 0, \theta_L = 0\}$ and

$$\left\{\theta_H = -\left(\frac{1}{4}\right)\frac{1011\Omega^3 - 4090\Omega^2 + 4632\Omega - 1408}{c(213\Omega^3 - 922\Omega^2 + 1008\Omega - 256)}, \theta_L = \frac{\Omega}{c}\right\}$$

where

$$\Omega = \text{Root of} \quad (128 - 584\chi + 836\chi^2 - 461\chi^3 + 84\chi^4)$$

The first two solutions are unfeasible (the first one has $\theta_H \ll \theta_L$ which has been ruled out by assumption); the third needs to investigate the root of the polynomial $128 - 584\chi + 836\chi^2 - 461\chi^3 + 84\chi^4$. Solving it w.r.t. χ yields two imaginary roots and two real solutions: $\chi_1 = 0.39872$, $\chi_2 = 2.71773$. Then if $\Omega = \chi_2$ we have $\theta_H = \frac{1.77763}{c}$ and $\theta_L = \frac{2.71773}{c}$, so that $\theta_H \ll \theta_L$, which has been ruled out by assumption. If instead $\Omega = \chi_1$ then $\theta_H^0 = \frac{0.81952}{c}$ and $\theta_L^0 = \frac{0.39852}{c}$. The latter is the solution of the pre–innovation quality game.

□

Proof of Proposition 2: From (27)–(29) and since $I_L^*(I_H = 0) = 1 \quad \forall\psi > 1$, we get Table 3, where, by inspection, the three equilibria emerge.

□

ψ range	$I_H(I_L=1)$	$I_H(I_L=0)$	$I_L(I_H=1)$	$I_L(I_H=0)$	Nash Eq.
$1 \leq \psi \leq 1.6110$	1	1	1	1	$I_H^* = I_L^* = 1$
$1.6110 < \psi \leq 1.6517$	1	0	1	1	$I_H^* = I_L^* = 1$
$1.6517 < \psi \leq 1.7509$	0	0	1	1	$I_H^* = 0, I_L^* = 1$
$1.7509 < \psi$	0	0	1	0	$I_H^* = I_L^* = 0$

Table 3: Best replies and Nash equilibria under Bertrand

Proof of Proposition 3: The two FOCs' (36)–(37) are simultaneously satisfied when

$$16\theta_H^2 - 4\theta_H\theta_L + 2\theta_L^2 - c(24\theta_H^3 - 10\theta_H^2\theta_L + 4\theta_H\theta_L^2 + \theta_L^3) = 0$$

and

$$8\theta_H - 2\theta_L + c(4\theta_H^2 - 23\theta_H\theta_L + 2\theta_L^2) = 0$$

The system has three solutions: $\{\theta_H = \frac{2}{7}\frac{1}{c}, \theta_L = \frac{8}{7}\frac{1}{c}\}$, $\{\theta_H = 0, \theta_L = 0\}$ and

$$\left\{\theta_H = \frac{1}{107}\frac{2993\Upsilon^2 - 995\Upsilon + 72}{c(182\Upsilon^2 - 79\Upsilon + 8)}, \theta_L = \frac{2}{c}\Upsilon\right\}$$

where

$$\Upsilon = \text{Root of} \quad (-16 + 126\xi - 463\xi^2 + 749\xi^3)$$

As in the proof of Proposition 1 the first two solutions are unfeasible while the third needs to investigate the root of the above polynomial. The unique real solution is $\xi = 0.29279$. Then we have $\theta_H^0 = \frac{0.73810}{c}$ and $\theta_L^0 = \frac{0.58558}{c}$.

□

Proof of Proposition 4: From (46)–(49), we get Table 4, and the indicated Nash equilibria.

□

ψ range	$I_H(I_L = 1)$	$I_H(I_L = 0)$	$I_L(I_H = 1)$	$I_L(I_H = 0)$	Nash Eq.
$1 \leq \psi \leq 1.5995$	1	1	1	1	$I_H^* = I_L^* = 1$
$1.5995 < \psi \leq 1.6036$	0	1	1	1	$I_H^* = 0, I_L^* = 1$
$1.6036 < \psi \leq 1.6485$	0	0	1	1	$I_H^* = 0, I_L^* = 1$
$1.6485 < \psi \leq 1.6636$	0	0	1	0	$I_H^* = I_L^* = 0$
$1.6636 < \psi \leq 2.8137$	0	0	0	0	$I_H^* = I_L^* = 0$
$1.6485 < \psi 2.8137$	0	0	1	0	$I_H^* = I_L^* = 0$

Table 4: Best replies and Nash equilibria under Cournot

References

- Athey, S.,- Schmutzler, A., 1995, Product and Process Flexibility in an Innovative Environment, *Rand Journal of Economics*, 26, 557–74.
- Battaggion, M.R.,- Tedeschi, P., 1998, Innovation and Vertical Differentiation, Working Paper n. 75, Università degli Studi di Padova, Padova.
- Bonanno, G., 1986, Vertical Differentiation with Cournot Competition, *Economic Notes*, 15, 68–91.
- —,- Haworth, B., 1998, Intensity of Competition and the Choice between Product and Process Innovation, *International Journal of Industrial Organization*, 16, 495–510.
- Champseaur, P.,- Rochet, J.C., 1989, Multiproduct Duopolists, *Econometrica*, 57, 533–557.
- Choi, C.J.,- Shin, H.S., 1992, A Comment on a Model of Vertical Product Differentiation, *Journal of Industrial Economics*, 40, 229–232.
- Gal–Or, E., 1983, Quality and Quantity Competition, *Bell Journal of Economics*, 14, 590–600.
- Ecchia, G.,- Lambertini, L., 1998, Market Coverage and the Existence of Equilibrium in a Vertically Differentiated Duopoly, Discussion Paper n. 311, Dipartimento di Scienze Economiche, Università di Bologna.
- Eswaran, M.,- Gallini, N., 1996, Patent Policy and the Direction of Technological Change, *Rand Journal of Economics*, 27, 722–746.
- Herguera, I.,- Lutz, S., 1998, Oligopoly and quality leapfrogging, *The World Economy*, 21, 75–94.
- Lambertini, L.,- Orsini, R., 2000, Process and product innovation in a vertically differentiated monopoly, *Economics Letters*, 68, 333–37.
- Lutz, S., 1997, Vertical Product Differentiation and Entry Deterrence *Journal-of-Economics-(Zeitschrift-fur-Nationalokonomie)*, 65, 79–102.

- Motta, M., 1993, Endogenous Quality Choice: Price vs Quantity Competition, *Journal of Industrial Economics*, 41, 113–131.
- —, Thisse J.F. and A. Cabrales, 1997, On the Persistence of Leadership or Leapfrogging in International Trade, *International Economic Review*, 38, 809–824.
- Mussa, M.,- Rosen, S., 1978, Monopoly and Product Quality, *Journal of Economic Theory*, 18, 301–317.
- Rosenkranz, S., 2003, Simultaneous Choice of Process and Product Innovation when Consumers have a Preference for Product Variety, *Journal of Economic Behavior and Organization*, 50, 183–201.
- Shaked, A.,- Sutton, J., 1982, Relaxing Price Competition through Product Differentiation, *Review of Economic Studies*, 49, 3–14,
- —, —, 1983, Natural Oligopolies, *Econometrica*, 51, 1469–1483.
- Tirole, J., 1988, *The Theory of Industrial Organization*, Cambridge, Mass., MIT Press.
- Wauthy, X., 1996, Quality Choice in Models of Vertical Differentiation, *Journal of Industrial Economics*, 44, 345–353.
- Weiß, P., 2002, Product and Process Innovation in a Horizontally Differentiated Product Market, Chemnitz University of Technology.