

# **Large computational financial modelling and optimization systems as the driver of performance for managing market risk**

Ronald Hochreiter

Department of Statistics and Decision Support Systems, University of Vienna

EUMOptFin 3 Workshop, Bergamo, Italy - May 17-21, 2004

# Outline

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- Introduction / Mapping and remapping the real world
- Handling real-world (financial) uncertainty: Stochastic programming
- AURORA Financial Management System
  - Component-based financial modelling and optimization, FMS users
- Numerical Examples: Mean-(Market)Risk portfolio optimization
  - Backfitting, scenario reduction, smart server performance
- Conclusion

# Introduction / Deducing optimal financial decisions

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- **Optimal financial decisions.** Optimization models highly depend on subjective algorithms based on financial managers beliefs.
- **Integrated financial management system.** From trader support (subjective algorithm orchestration) to client reporting (webtops).
  - FMS offers possibilities to match the needs of the industry (see example)
- **Quality of Service/Efficiency.** FMS with optimization models grant QoS
  - Argumentability, responsibility, post-trade analysis → efficiency measurement

# Mapping and remapping the real world

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- Map the real world problem into a numerical model.
- Approximate real-world data for numerical computation on computers.
  - Data is uncertain and/or inexact (approximation/extrapolation needed).
- Solve resulting (optimization) model.
  - Models of real-world problems are large-scale.
- Deduce a decision from the solution of the model.
  - Remap the numerical solution to the real world (interpretation/implementation).

# AURORA - Optimization and Modelling

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- **SRP AURORA** - Advanced Models, Applications and Software Systems for High Performance Computing / High Performance Computing in Finance.
- **Optimization** under uncertainty (for finance) mainly driven by results from computational optimization and high performance computing.
- **Modelling** via component analysis, decomposition, orchestration and enactment - Grid (P2P) and Web technologies.
  - Workflow Management Technology, abstract Open Grid Service Environment (OGSE)

# AURORA - Financial Optimization and Modelling

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- **Strategical level.** Multi-stage stochastic programming models (long term [investment](#) horizon, small number of asset classes (risky stocks, stocks, bonds, cash, property))
- **Tactical level.** Single-stage robust stochastic programming models (mean-risk portfolio selection), time horizon 1m-1y, e.g. define industry sectors for stock investments.
- **Operational level.** Smart Servers, applying (semi-)automatic optimization for intra-day [trading](#) with a huge number of stocks.

# Workflow decomposition/enactment environment

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(Completely) different modelling and optimization techniques for investment and trading level necessary. AURORA Financial Management System inherited basic ideas workflow management system:

**Analyze.** Analyze and decompose (algorithmic) financial engineering workflows.

**Collect.** Build collection of various implementations (algorithms) for different tasks within financial workflows.

**Compare.** Compare different implementations of subjective algorithms in an unified framework → semi-automatic selection.

# Stochastic programming - Overview [1/2]

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Consider the classical deterministic optimization problem:  $\min\{f(x)|x \in \mathcal{X}\}$

- Stochastic optimization: uncertain variables (e.g. stock prices, ...) in a model are described as random variables, whose probability distributions are assumed to be known (or can be estimated in a way or another).
- Maximize (or minimize) expectation (or some other functional).
- Solutions (decisions) are well-hedged against all (bad) outcomes.

# Stochastic programming - Overview [2/2]

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Deterministic cost function  $f(x)$  replaced with a stochastic one  $f(x, \xi)$  and the expectation functional of this stochastic cost function is minimized:

$$\min\{\mathbb{E}(f(x, \xi)) | x \in \mathcal{X}\}$$

Finance: Mean-Deviation (Mean-Risk) functionals (see later)

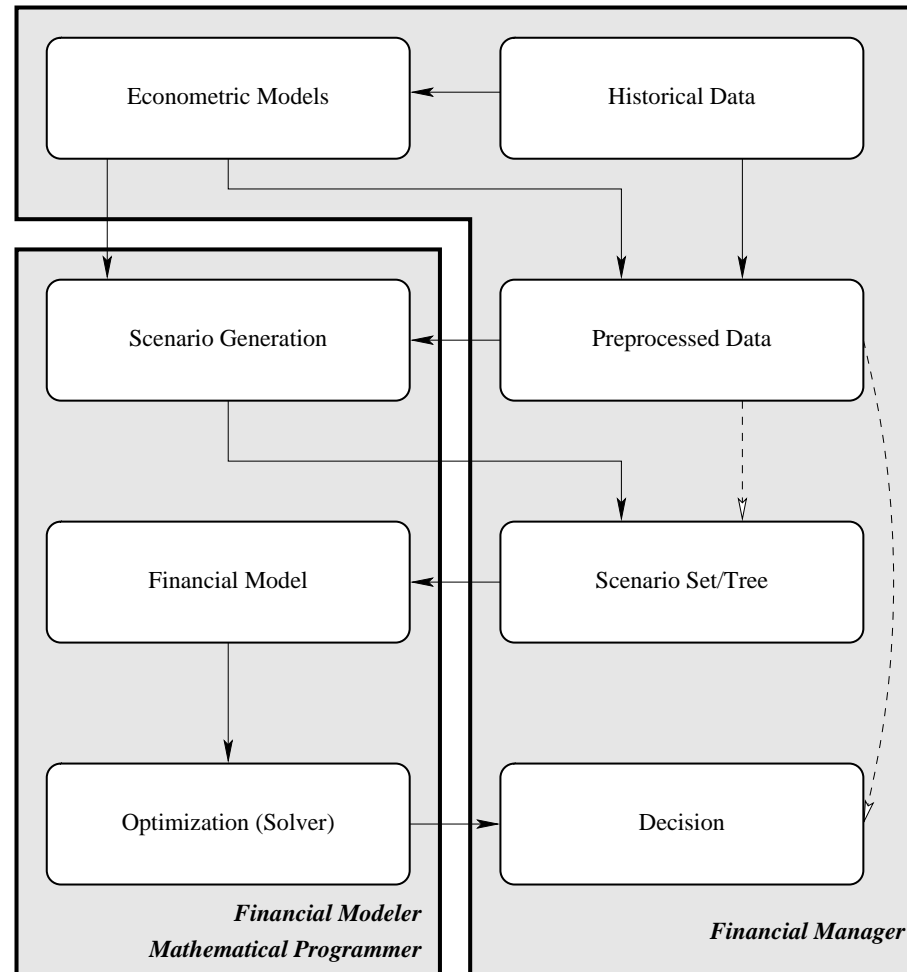
$$\min\{\mathbb{F}(f(x, \xi)) | x \in \mathcal{X}\}$$

$$\mathbb{F}(f(x, \xi)) := -\mathbb{E}(f(x, \xi)) + \lambda \mathbb{D}(f(x, \xi))$$

Define time-discrete multi-stage problems with recourse:  $x \rightsquigarrow \xi(x) \rightsquigarrow y$

# Deducing financial decisions: Optimization Process

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# Stochastic programming - Scenario generation

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Scenario optimization/generation/reduction: Discrete approximation of  $\xi \rightarrow \hat{\xi}$

$$\text{minimize}\{\hat{f}_N(x) = \mathbb{F}(x, \hat{\xi}) : x \in \mathcal{X}\}$$

Reduce the complexity of data to numerically solve the problem.

**Optimization problem-related scenario optimization - goals:**

- approximation of objective function  $\sup_{x \in \mathcal{X}} |\hat{f}_N(x) - f(x)|$ ,
- approximation of the argmin:  $\|\text{argmin}_{x \in \mathcal{X}} \hat{f}_N(x) - \text{argmin}_{x \in \mathcal{X}} f(x)\|$

# Which scenario generation model to choose?

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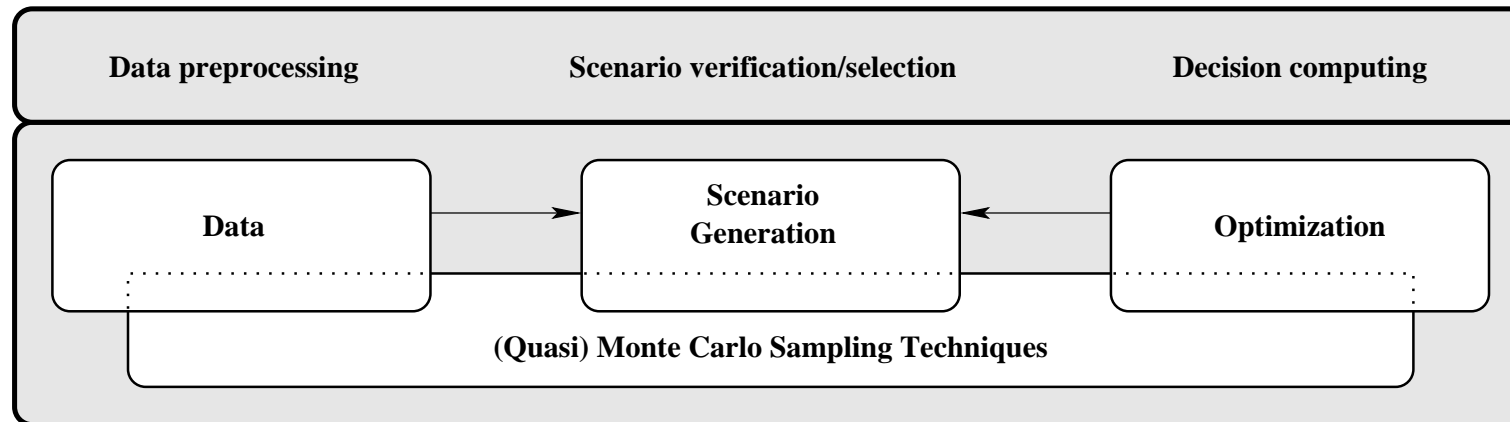
**S.W. Wallace [ISMP 2003]:** Hide the scenario model from the user. (Remark: Of course, if you want to sell it, hide the math)

**Another possible extreme:** Grant financial manager the opportunity to choose her *favorite* scenario generation technique (accentuate that *model risk* induced in choosing scenario model). Furthermore,

- Scenario generation methodology mixtures (one for discretization and one for fitting to optimization problem).
- Moving *model logic* from the optimization model to the scenario generation procedure.

# Different approaches to scenario generation

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- Høyland/Wallace: Plainly match first four moments (investors choice?)
- Pflug & Römisch: Optimization problem-related scenario optimization
- Pennanen/Koivu: (R)QMC sampling (optimization problem-related)

# Main motivation for setting up a new FMS (1)

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- Many different (isolated, heterogeneous) implementations of subjective financial algorithms exist (scenario generation methodologies for stochastic programming)
- Nearly impossible to conduct an unbiased comparison
- Solution due to tight collaboration with computer science (AURORA)
  - General implementation: workflows on an abstract layer - OGSE
  - Communication (data formats, semantics): XML, MathProgML, ...

# FMS - Different classes and views of users [1/2] (2)

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Various classes of users of financial management systems:

- Financial manager (user)
- Financial engineer, component engineer (developer)
- Mathematical programmer, econometrician (researcher)

**Managers view:** problem solving ((semi-)automatic component orchestration)

**Developers view:** workflow communication/specification, component wrappers

**Researchers view:** algorithmic bottleneck analysis, ...

# FMS - Different classes and views of users [2/2]

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Various classes of users of financial management systems:

- Financial manager (user)
- Financial engineer, component engineer (developer)
- Mathematical programmer, econometrician (researcher)
- Joanna and John Doe (customers)

**Customers view:** e.g. how well is my unit-linked life-insurance performing ((web-based) client reporting)

# Financial models - the need of the industry

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**Need of the industry.** Stavros Siokos, Citigroup [CMS 2004]:

1. Quantitative accuracy (real-life assumptions, uncertainty, accurate back test)
2. Perception of reliability (test against internal data, no time and expense on theoretical models)
3. Analytical simplicity (simplicity, transparency, multi-assets)

**Smart Servers.** intelligent trading, auto-executing trades, pre-determined trading strategy

# Market risk modelling: Mean-Risk optimization

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Deducing optimal financial investment decisions. Decision vector  $x_i$  relative amount of budget to invest in asset (class)  $i$  out of  $n$  possible asset( classe)s.

Motivated by Nobel-prize honored ideas of H.M. Markowitz [1952, 1959]:

$$\begin{aligned} & \text{maximize} && \mathbb{E}(f(x, \xi)) - \lambda \mathbb{D}(f(x, \xi)) \\ & \text{subject to} && x \in \mathcal{X} \end{aligned} \tag{1}$$

Examples for  $\mathbb{D}(\cdot)$ : Variance (Markowitz 1952), Gini Mean Difference (Yitzhaki 1982), MAD (Konno/Yamazaki 1991), CVaR (Rockafellar/Uryasev 2000), ...

$\mathcal{X}$  - (dis)allow shortselling, regulatory/organizational constraints, ...

Single-stage: reformulation to linear or quadratic programs possible.

# Example: Mean-Risk portfolio optimization

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## Dow Jones STOXX Market Sector Indexes (Europe)

**Considering all 18 sectors:** Automobiles, Banks, Basic Resources, Chemicals, Construction, Cyclical Goods & Services, Energy, Financial Services, Food & Beverage, Healthcare, Industrial Goods & Services, Insurance, Media, Non-cyclical Goods & Services, Retail, Technology, Telecom, Utilities

**Eleven years of daily data:** January 1st, 1992 to December 30th, 2002

**Examples:** simple backfitting, scenario reduction and backfitting.

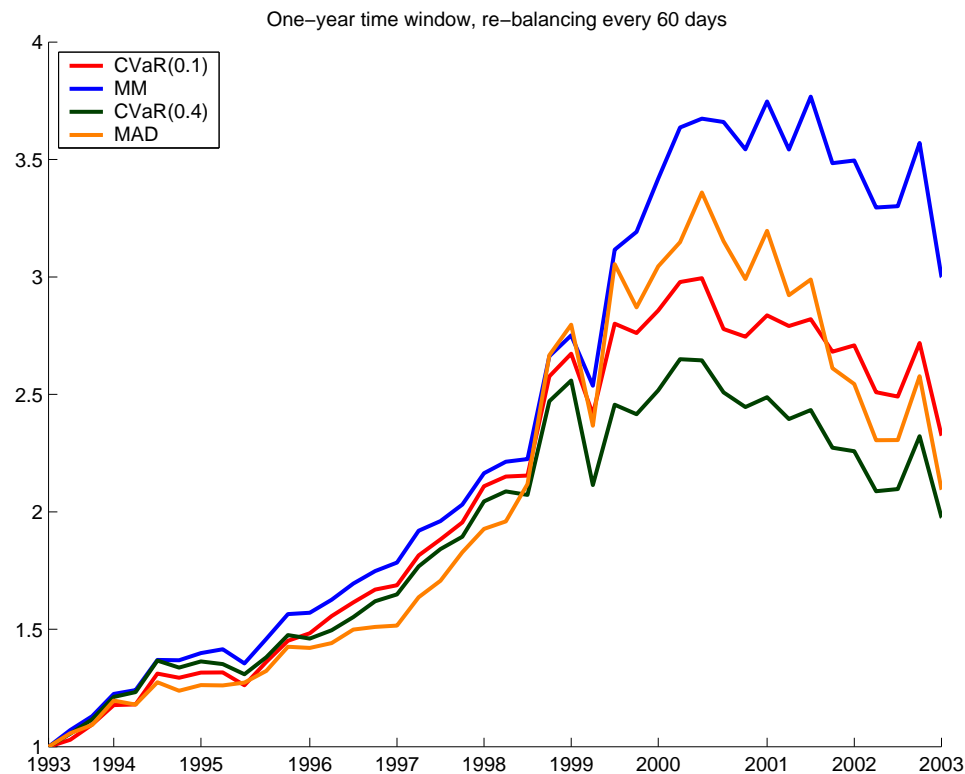
**Futhermore:** Backfitting-based automatic risk measure selection for intra-day trading?

# Example One: Simple backfitting

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Comparing  $\mathbb{D}$  (LP): MAD, MiniMax,  $\text{CVaR}_{\alpha=0.1}$ ,  $\text{CVaR}_{\alpha=0.4}$

One-year time window, portfolio rebalancing every 60 days.



## Example Two: Scenario reduction

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- Moment matching ( $M = 4$ ) (Høyland et al. 2003)

$$d_{MM}(P, Q) = \sup\left\{\int w^p dP(w) - \int w^p dQ(w) : 1 \leq p \leq M\right\}$$

- Probability distance minimization (Wasserstein  $p = 1$  (Pflug 2001))

$\mathcal{H}$  ... class of Lipschitz continuous functions of order  $p$ :

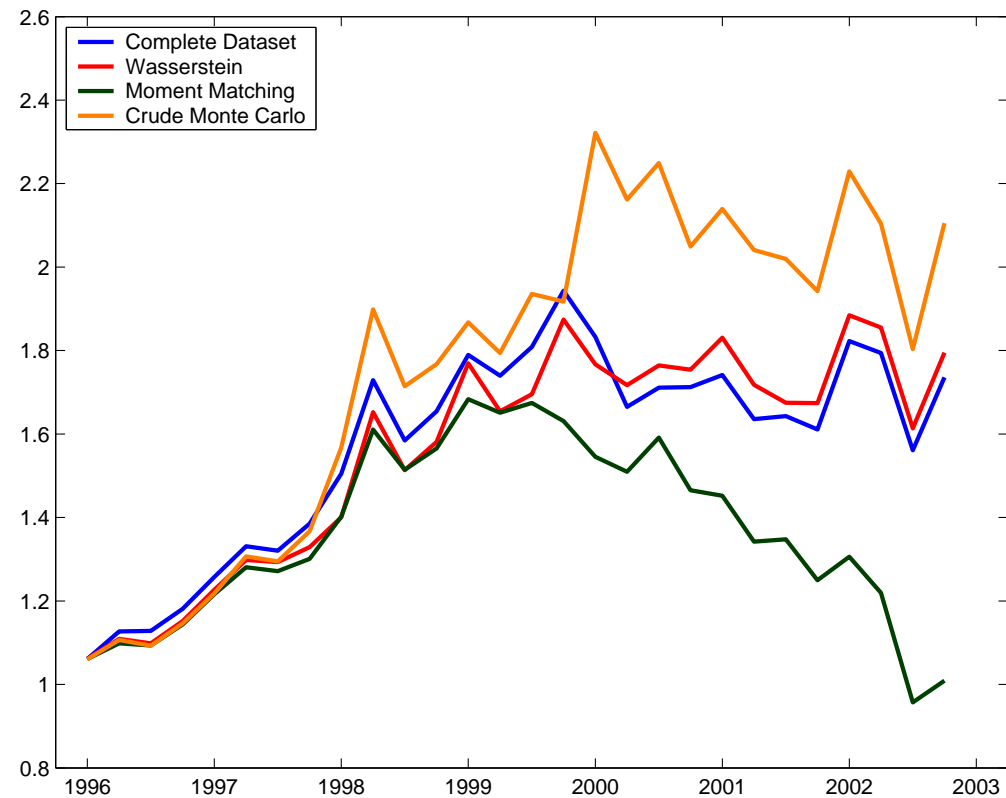
$$d_{\mathcal{H}}(P, Q) = \sup\left\{\left|\int h(x, w) dP(w) - \int h(x, w) dQ(w)\right| : h \in \mathcal{H}\right\}$$

- Quasi-random sampling (Low discrepancy series: Sobol sequence)

# Scenario reduction and backfitting

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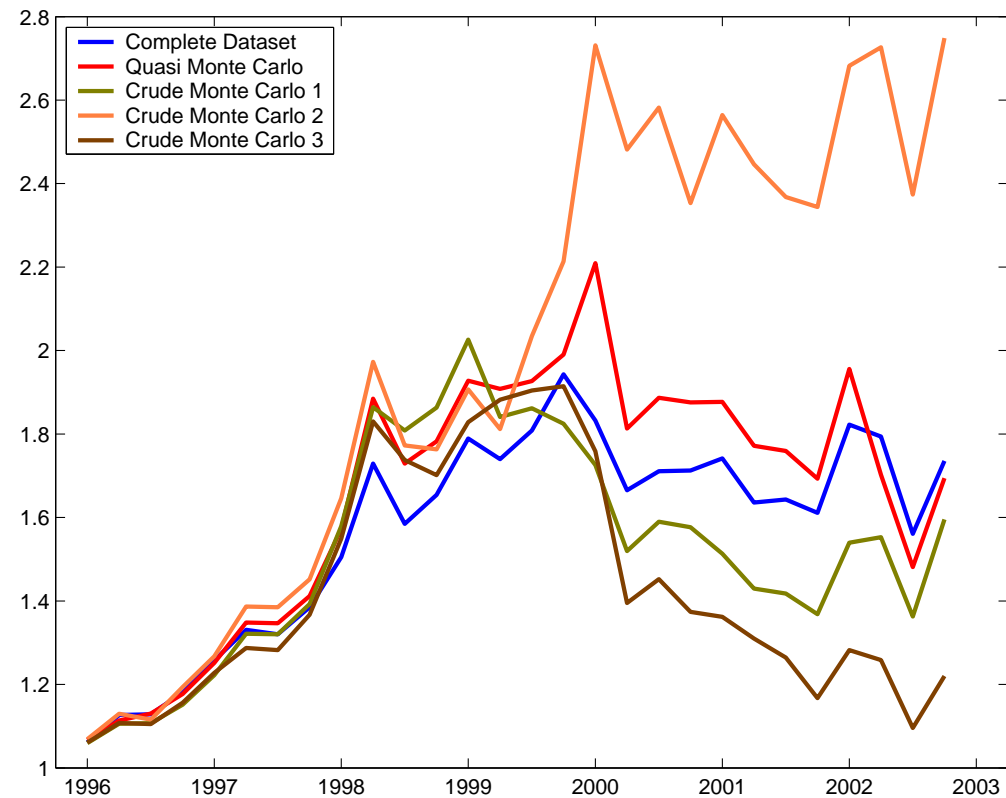
Conditional Value at Risk (Expected Shortfall)  $\alpha = 0.4$ ,  $|\xi| \simeq 500$ ,  $|\hat{\xi}| = 50$



# Scenario reduction and backfitting

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Conditional Value at Risk (Expected Shortfall)  $\alpha = 0.4$ ,  $|\xi| \simeq 500$ ,  $|\hat{\xi}| = 50$



## Example 3: Backtesting-reliable automatic trading

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Workflow based system ideal framework to conduct backtesting with different subjective algorithms and parameters. Finding the needle in the haystack.

**Automatic optimization system.** Set of mean-risk models (strategies) available

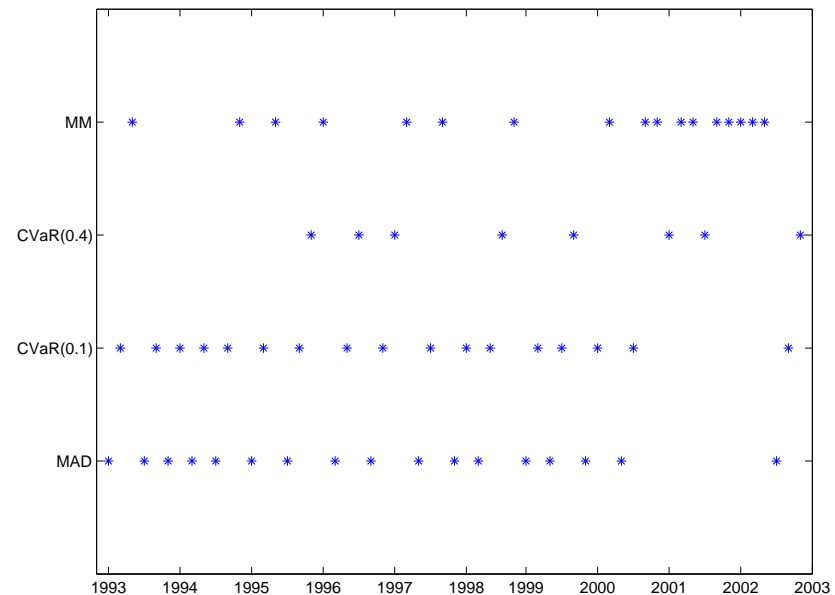
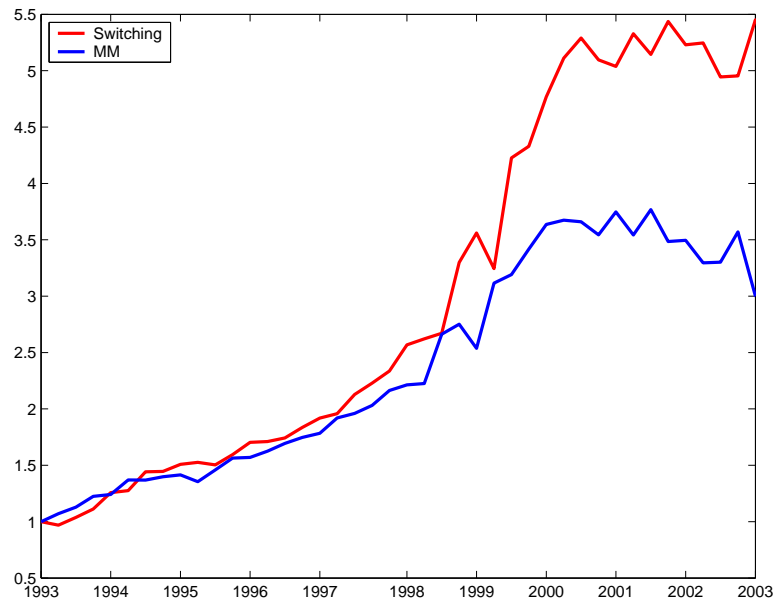
- Decision which strategy to use based on short-term backtesting
- Use a mixture of historical data and forecasts.
- Define switching rules. Only switch strategy, e.g. if  $\hat{r}_{new} \geq \delta \hat{r}_{old}, \dots$

# Example 3: Backtesting-reliable automatic trading

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Comparing  $\mathbb{D}$  (LP): MAD, MiniMax,  $\text{CVaR}_{\alpha=0.1}$ ,  $\text{CVaR}_{\alpha=0.4}$

One-year time window, portfolio rebalancing every 60 days, backlook: 60 days, no additional switching strategy, plain historical data.



# Financial models - the need of the industry *revised*

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**Need of the industry.** Stavros Siokos, Citigroup [CMS 2004]:

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# Conclusion

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- Once an abstract environment (OGSE) has been set up, different (financial) applications for various operational levels can be enacted quickly and successfully to fulfill the needs of the industry.
- EUMOptFin3: Integrated efficiency measurement, the set of user classes of the FMS (manager, developer, researcher) has been extended with the customer.
- Semi-automatic optimization model selection trading - a valuable approach?

# AURORA - High Performance Computing in Finance

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AURORA - Advanced Models, Applications and Software Systems for High Performance Computing

<http://www.vcpc.univie.ac.at/aurora/>

AURORA - Sub-Project Computational Finance:

<http://www.compmath.net/aurora/>