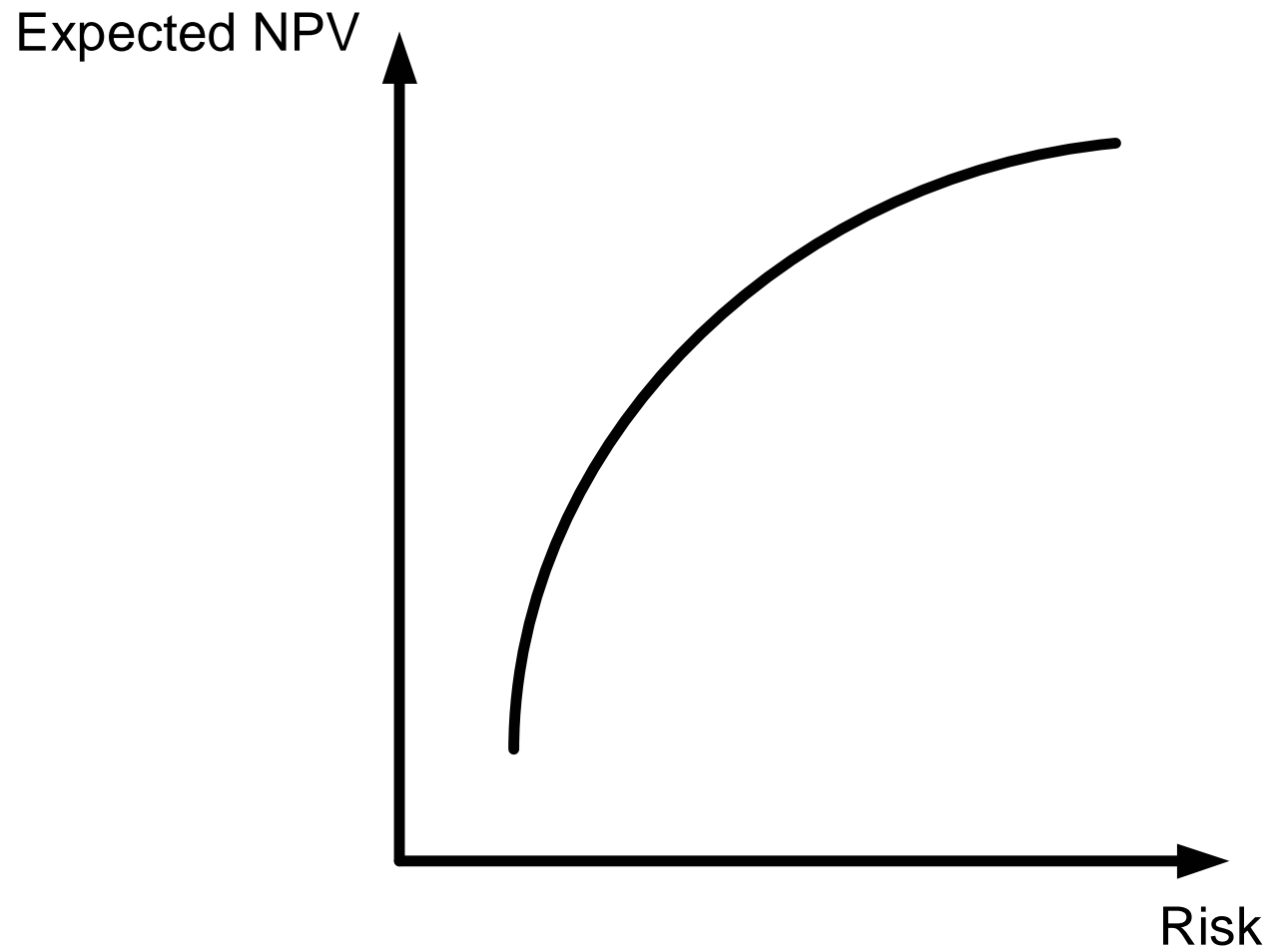


Mean-Risk Analysis in Business Portfolio Selection with Capital Constraints

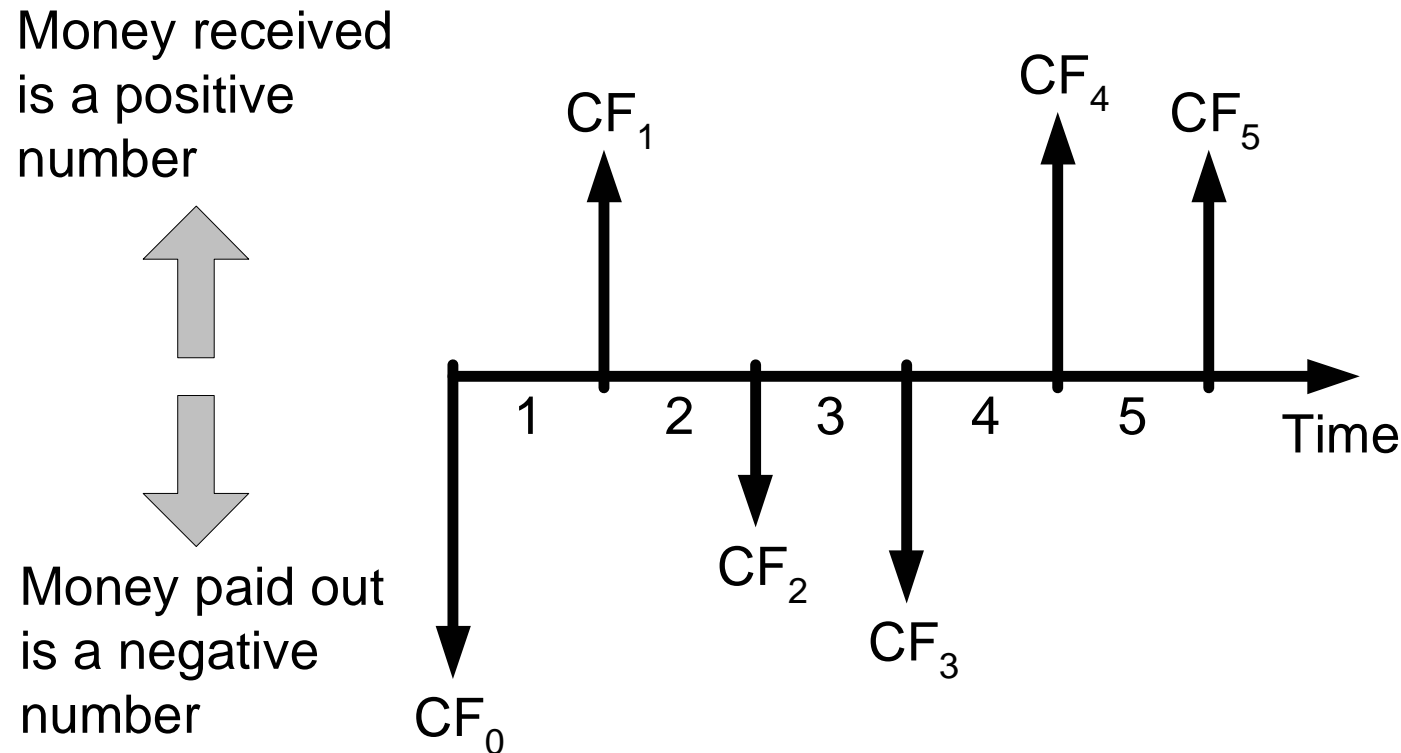
Adam Krzemienowski

Institute of Control and Computation Engineering
Warsaw University of Technology

General framework



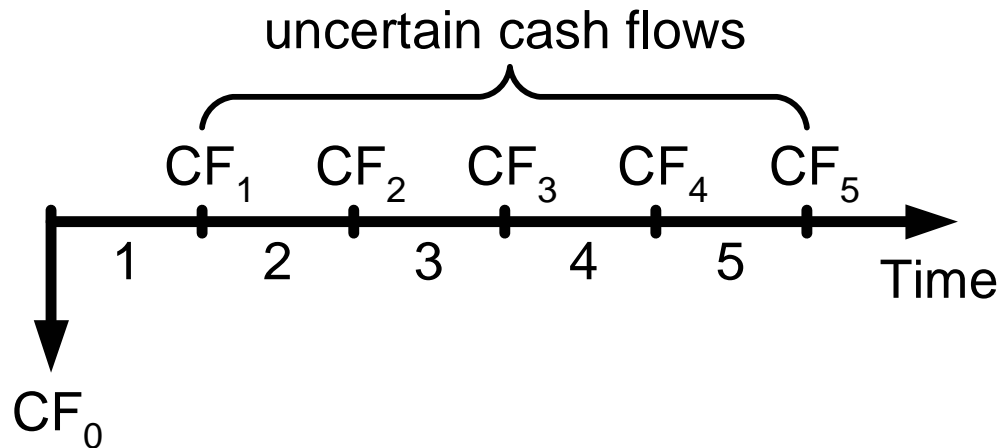
NPV concept



$$NPV = \sum_{i=0}^n PV_i = \sum_{i=0}^n \frac{CF_i}{(1+r)^i}$$

where r is the **rate of return on an alternative investment**

Expected NPV of a single business line



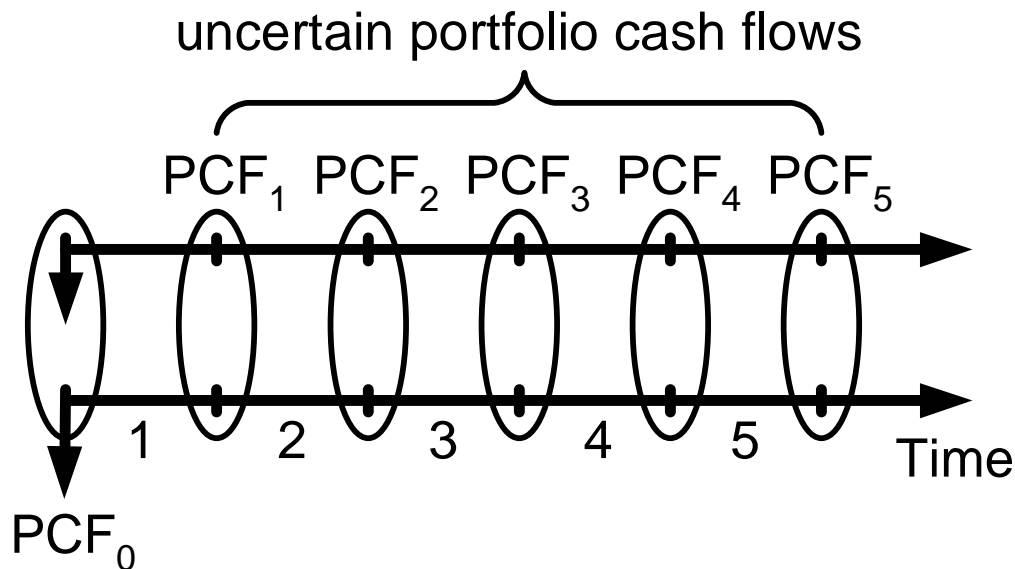
Distribution of CF_1 :

$$CF_1 = \begin{cases} A, & p = a \\ B, & p = b \end{cases}$$

$$\mathbb{E}\{\text{NPV}\} = \sum_{i=0}^n \mathbb{E}\{\text{PV}_i\} = \sum_{i=0}^n \frac{\mathbb{E}\{CF_i\}}{(1 + \text{WACC})^i}$$

where WACC is the **Weighted Average Cost of Capital**

Expected NPV of multiple business lines



Joint distribution of PCF₁:

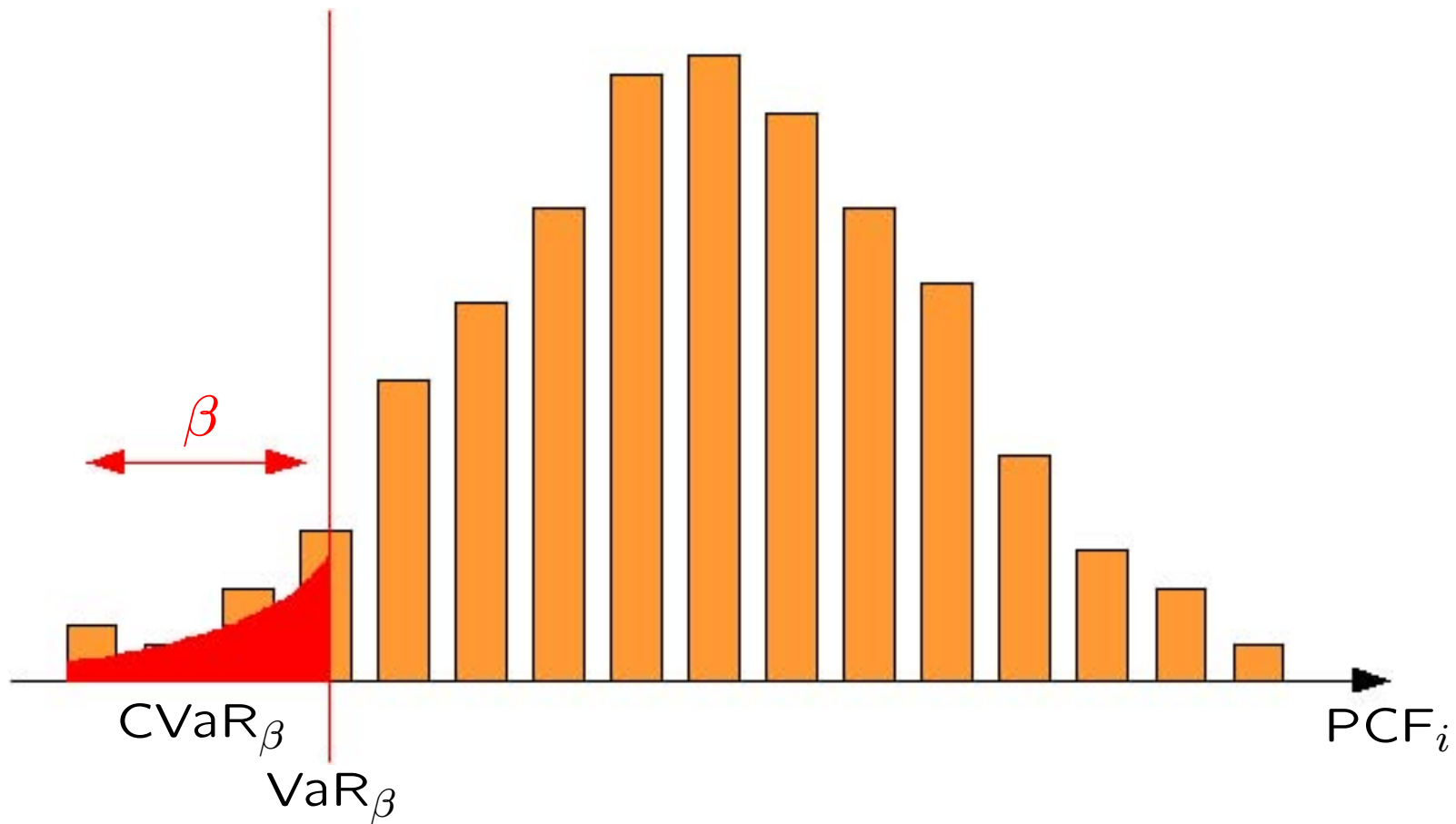
$$PCF_1 = \begin{cases} A_1 + A_2, & p = a_1 a_2 \\ A_1 + B_2, & p = a_1 b_2 \\ B_1 + A_2, & p = b_1 a_2 \\ B_1 + B_2, & p = b_1 b_2 \end{cases}$$

$$\mathbb{E}\{\text{NPV}\} = \sum_{i=0}^n \mathbb{E}\{\text{PV}_i\} = \sum_{i=0}^n \frac{\mathbb{E}\{\text{PCF}_i\}}{(1 + \text{WACC})^i}$$

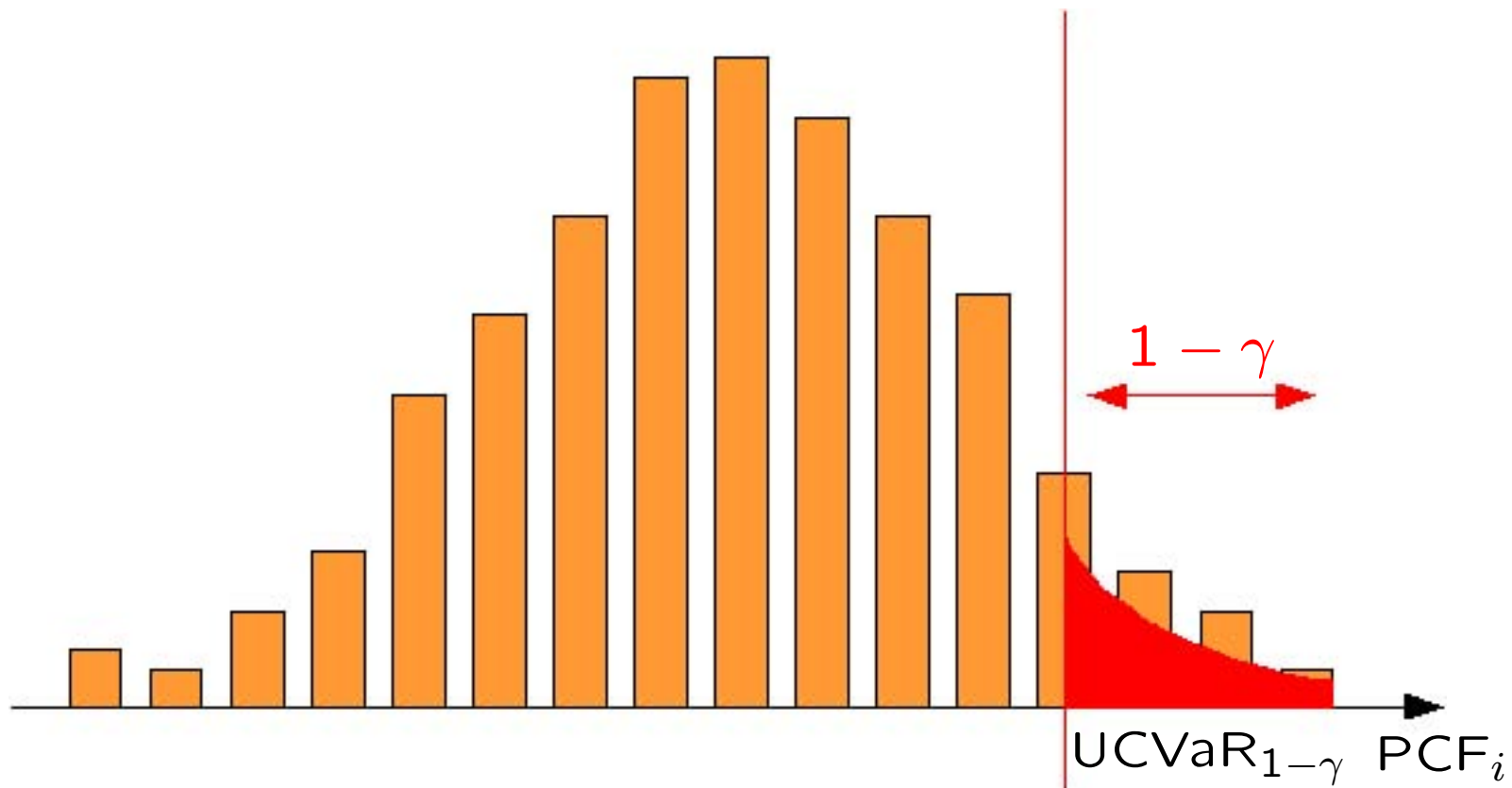
Risk

- Handled by Conditional Average (CAVG)
- CAVG is a combination of Conditional Value-at-Risk (CVaR) and Upside Conditional Value-at-Risk (UCVaR)
- CAVG covers all attitudes towards risk
- CAVG can be expressed as a quadratic optimization problem that can be further approximated by linear programming techniques

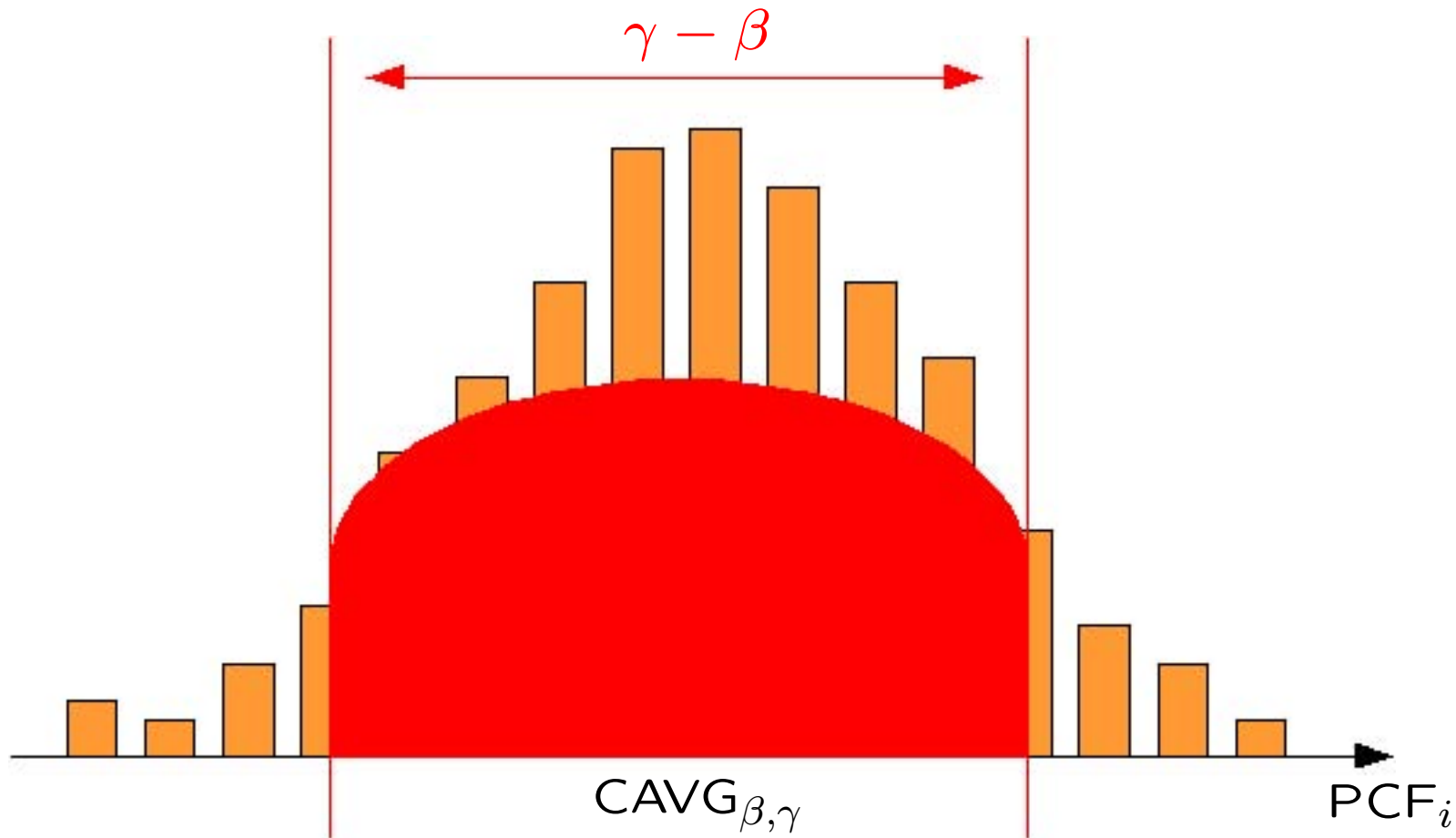
Conditional Value-at-Risk



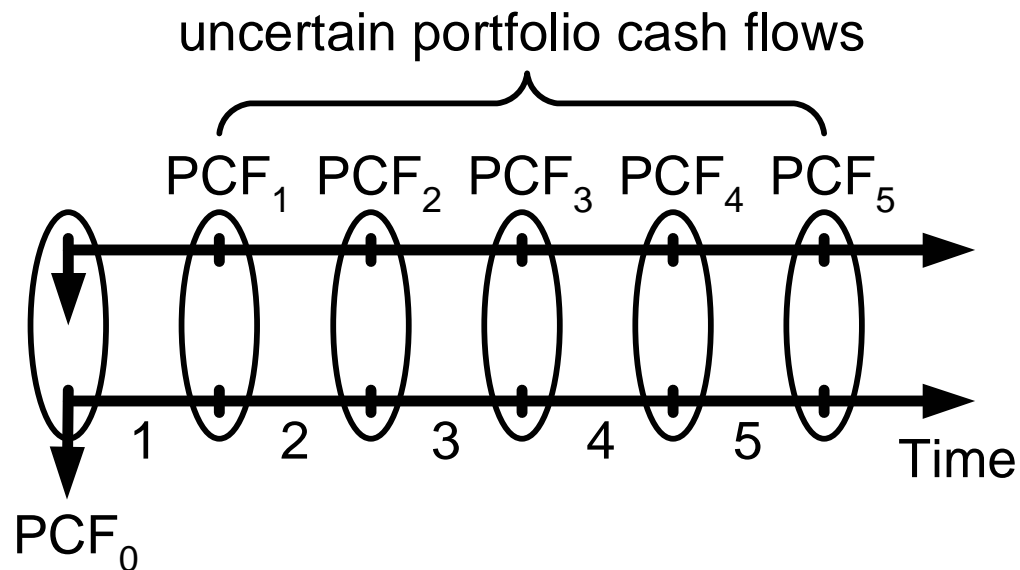
Upside Conditional Value-at-Risk



Conditional Average

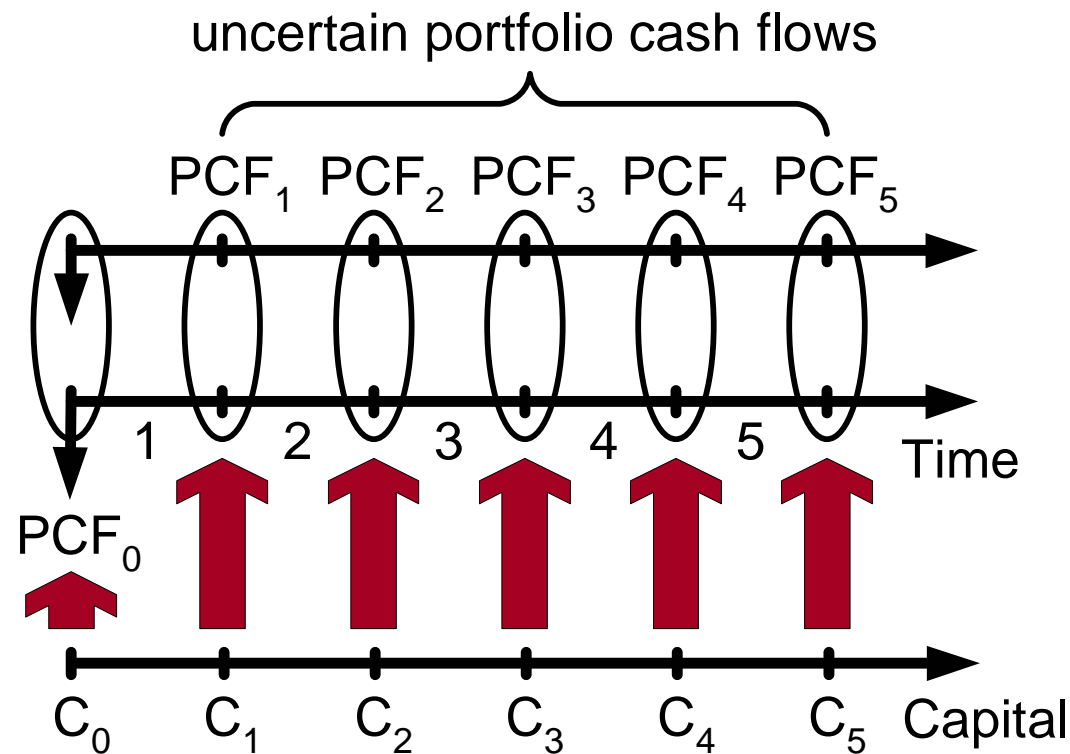


Risk of multiple business lines



$$\text{CAVG}_{\beta,\gamma}(\text{NPV}) = \sum_{i=0}^n \text{CAVG}_{\beta,\gamma}(\text{PV}_i) = \sum_{i=0}^n \frac{\text{CAVG}_{\beta,\gamma}(\text{PCF}_i)}{(1 + \text{WACC})^i}$$

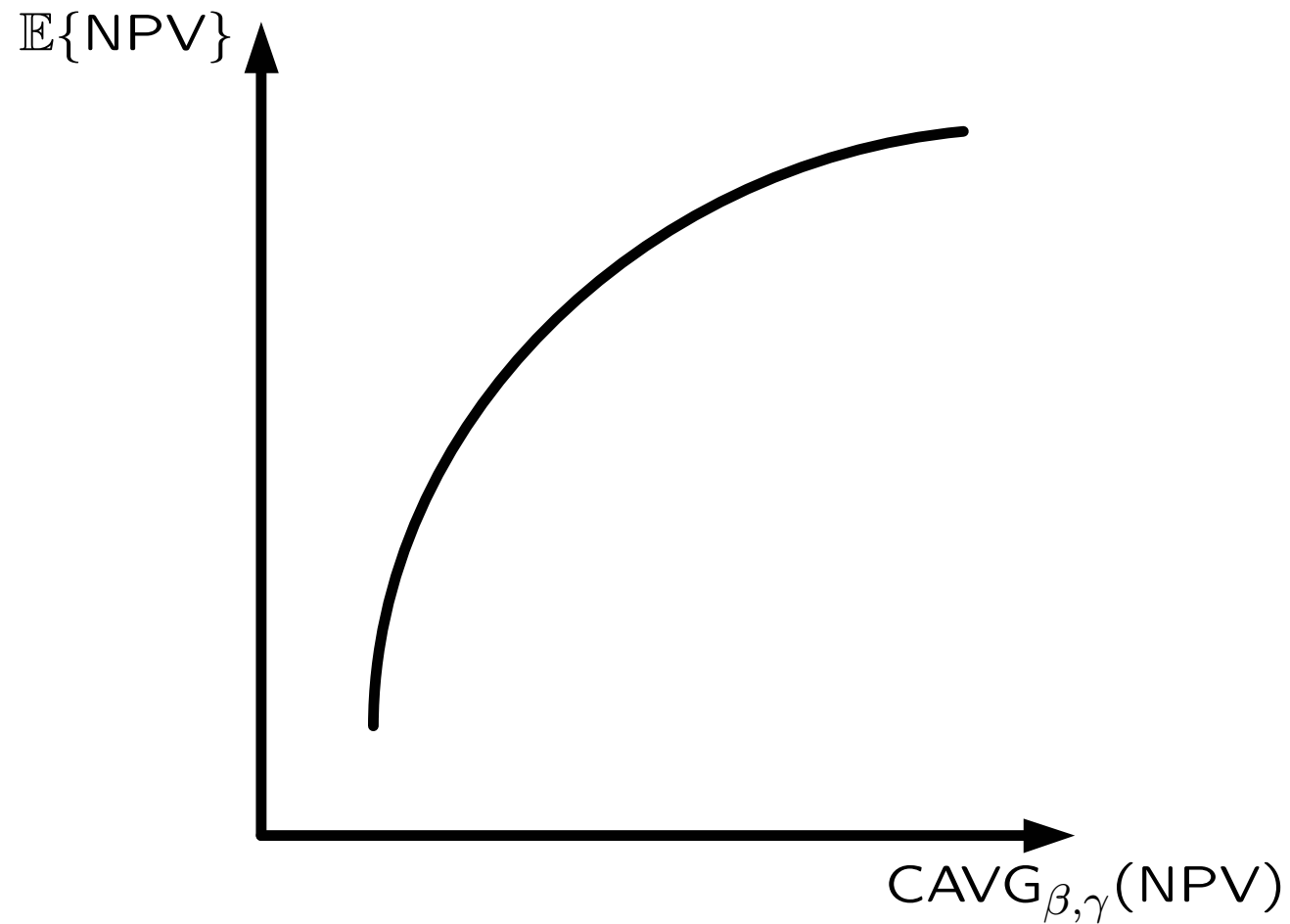
Capital constraints



Portfolio inflows cannot exceed the available amount of capital:

$$PCF_0 + C_0 \geq 0 \quad \text{and} \quad \text{CAVG}_{\beta, \gamma}(PCF_i) + C_i \geq 0 \quad \text{for } i = 1, \dots, n$$

Business portfolio optimization framework



Business portfolio optimization model

$$\begin{aligned} \max \quad & \sum_{i=0}^n \frac{\text{CAVG}_{\beta,\gamma}(\text{PCF}_i(\mathbf{x}))}{(1 + \text{WACC})^i} \\ \text{s.t.} \quad & \sum_{i=0}^n \frac{\mathbb{E}\{\text{PCF}_i(\mathbf{x})\}}{(1 + \text{WACC})^i} \geq \text{required NPV} \\ & \text{PCF}_0 + C_0 \geq 0 \\ & \text{CAVG}_{\beta,\gamma}(\text{PCF}_i(\mathbf{x})) + C_i \geq 0 \quad \text{for } i = 1, \dots, n \\ & \mathbf{x} \in \{0, 1\} \text{ — business portfolio} \end{aligned}$$