Mean-Risk Analysis in Business Portfolio Selection with Capital Constraints

Adam Krzemienowski

Institute of Control and Computation Engineering
Warsaw University of Technology
General framework
NPV concept

Money received
is a positive
number

Money paid out
is a negative
number

NPV = \sum_{i=0}^{n} PV_i = \sum_{i=0}^{n} \frac{CF_i}{(1 + r)^i}

where \( r \) is the rate of return on an alternative investment
Expected NPV of a single business line

Distribution of CF$_1$:

CF$_1 = \begin{cases} 
A, & p = a \\
B, & p = b 
\end{cases}$

$\mathbb{E}\{\text{NPV}\} = \sum_{i=0}^{n} \mathbb{E}\{PV_i\} = \sum_{i=0}^{n} \frac{\mathbb{E}\{CF_i\}}{(1 + WACC)^i}$

where WACC is the Weighted Average Cost of Capital
Expected NPV of multiple business lines

uncertain portfolio cash flows

PCF<sub>1</sub> PCF<sub>2</sub> PCF<sub>3</sub> PCF<sub>4</sub> PCF<sub>5</sub>

Time

Joint distribution of PCF<sub>1</sub>:

\[
PCF_1 = \begin{cases} 
A_1 + A_2, & p = a_1a_2 \\
A_1 + B_2, & p = a_1b_2 \\
B_1 + A_2, & p = b_1a_2 \\
B_1 + B_2, & p = b_1b_2 
\end{cases}
\]

\[
\mathbb{E}\{NPV\} = \sum_{i=0}^{n} \mathbb{E}\{PV_i\} = \sum_{i=0}^{n} \frac{\mathbb{E}\{PCF_i\}}{(1 + WACC)^i}
\]
Risk

- Handled by Conditional Average (CAVG)

- CAVG is a combination of Conditional Value-at-Risk (CVaR) and Upside Conditional Value-at-Risk (UCVaR)

- CAVG covers all attitudes towards risk

- CAVG can be expressed as a quadratic optimization problem that can be further approximated by linear programming techniques
Conditional Value-at-Risk
Upside Conditional Value-at-Risk

$1 - \gamma$

$\text{UCVaR}_{1-\gamma}$ $\text{PCF}_i$
Conditional Average

\[ \gamma - \beta \]

\[ \text{CAVG}_{\beta, \gamma} \]

\[ \text{PCF}_i \]
Risk of multiple business lines

uncertain portfolio cash flows

PCF_0
Time 1 2 3 4 5

CAVG_{\beta,\gamma}(NPV) = \sum_{i=0}^{n} CAVG_{\beta,\gamma}(PV_i) = \sum_{i=0}^{n} \frac{CAVG_{\beta,\gamma}(PCF_i)}{(1 + WACC)^i}
Portfolio inflows cannot exceed the available amount of capital:

\[ PCF_0 + C_0 \geq 0 \quad \text{and} \quad CAVG_{\beta,\gamma}(PCF_i) + C_i \geq 0 \text{ for } i = 1, \ldots, n \]
Business portfolio optimization framework

$E\{NPV\}$

$CAVG_{\beta,\gamma}(NPV)$
Business portfolio optimization model

\[
\max \sum_{i=0}^{n} \frac{\text{CAVG}_{\beta,\gamma}(\text{PCF}_i(x))}{(1 + \text{WACC})^i} \\
\text{s.t.} \sum_{i=0}^{n} \frac{\mathbb{E}\{\text{PCF}_i(x)\}}{(1 + \text{WACC})^i} \geq \text{required NPV} \\
\text{PCF}_0 + C_0 \geq 0 \\
\text{CAVG}_{\beta,\gamma}(\text{PCF}_i(x)) + C_i \geq 0 \quad \text{for } i = 1, \ldots, n \\
x \in \{0, 1\} \quad \text{— business portfolio}
\]