Utilising basket products to hedge credit risk

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What is Credit Risk ?

Credit Risk can be seen as the possibility that an unexpected variation of the credit quality of a subject, with respect to which a credit exposure exists, creates a corresponding unexpected variation of the market value of the credit exposition.

There exist different typologies : *insolvency risk ;*
migration risk .

The credit risk components :

- ✓ Expected loss: it is the loss that a lender expects to achieve with respect to a given credit portfolio.

$$EL_{t1,t2} = p_{t1,t2} \cdot (1 - RR)$$

- ✓ Unexpected loss: it represents the measure of the variability of the loss rate with respect to the expected value.

Instruments to manage the risk

Currently the market of the derivatives offers numerous products to carry out hedging strategies calls Credit Derivatives. These are over-the-counter instruments that allow to:

- ✓ to separate;
- ✓ to confer a price;
- ✓ to transfer the credit risk implied in each credit-exposure.

Main contract typologies :

✓ against default	**Credit default products***;
	Basket products*;
✓ against downgrading risk	Credit spread products;
	Total rate of return swaps.

* these are instruments that provide insurance against a particular company (for the CDS) or against two or more companies (for the basket) defaulting on its debt. The company is known as the *reference entity* and a default by the company is known as a *credit event*. The buyer of the protection (the *protection buyer*) makes periodic payments to the seller of protection (the *protection seller*) at a predetermined fixed rate per year. The payments continue until the end of the life of the contract or until a credit event, whichever is earlier.

Evaluation of CDS using Hull & White model

Now we can evaluate a CDS contract looking at the future outcomes and incomes of the *protection buyer* :

✓ Protection cost $\omega \cdot \sum_{i=0}^{T} p_i \cdot u(i) + \omega \cdot \pi \cdot u(T)$

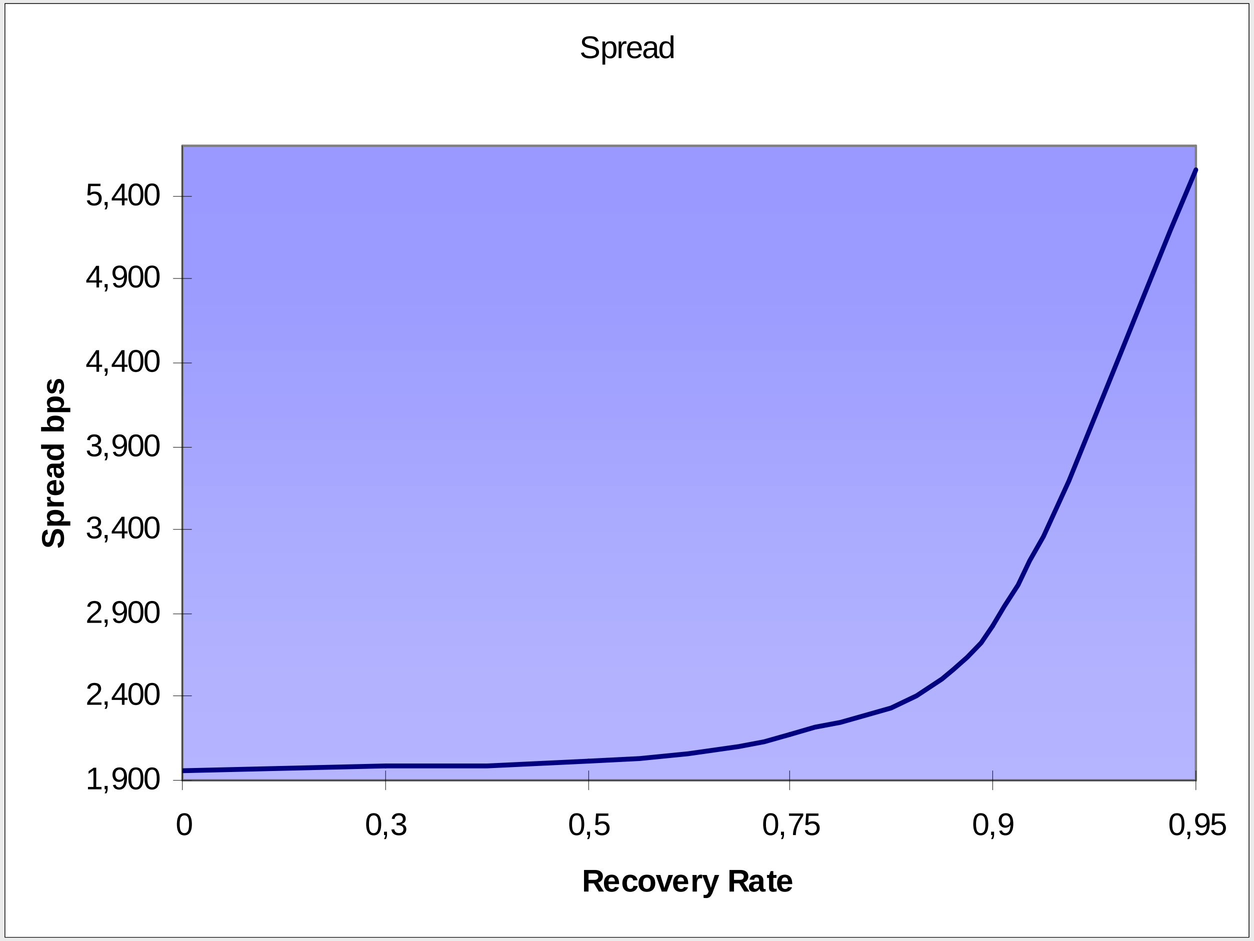
✓ Value of the protection $\sum_{i=0}^{T} (1-\hat{R}) \cdot p_i \cdot v(i)$

we obtain *w* such that the two values will coincide

i.e.

$$\omega = \frac{\sum_{i=0}^{T} (1-\hat{R}) \cdot p_i \cdot v(i)}{\sum_{i=0}^{T} p_i \cdot u(i) + \pi \cdot u(T)}$$

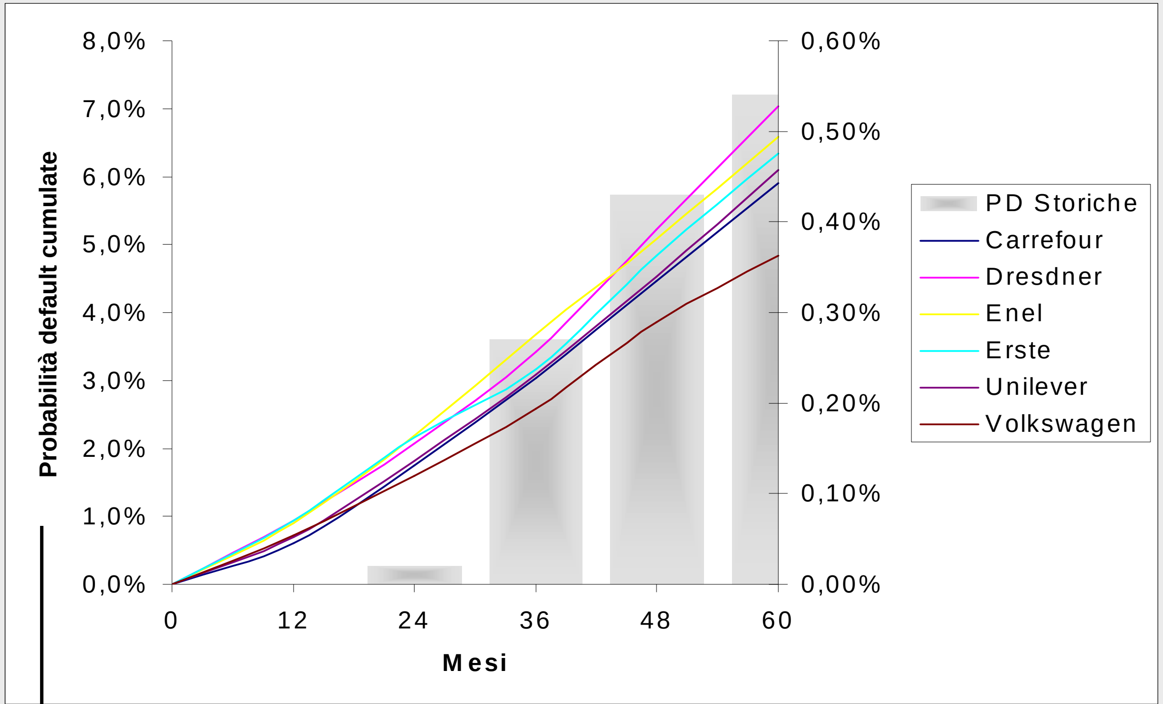
Hull & White model: remarks



Hull & White model: implementation

✓ *Reference entities* in euro-area issuer of corporate bonds with rating A1:

- Carrefour S.A.
- Enel SpA
- Unilever NV
- Dresdner Bank A.G.
- Erste Bank
- Volkswagen Finance



Risk-neutral default probabilities computing using market prices data of bonds based on Hull & White approach.

Hull & White model: implementation

Let assume to have a portfolio in which there is one bonds of each *reference entities* considered and let assume we want to carry out an hedging strategy using *credit default swaps* with an horizon time of 5 years which will protect the total investment from defaults: in this case we will have a total cost of 3.729.780 euro.

Reference Entities	*Spread in bps*	*Spread in euro (1.000)*
Carrefour S.A.	72,06928969	720,693
Dresdner Bank A.G.	86,20980168	862,098
Enel SpA	81,02908642	810,291
Erste Bank	77,8027794	778,028
Unilever NV	74,46061246	744,606
Volkswagen Finance	59,20932894	592,093

Table 1 : Spreads of a CDS at 5 years.

Source: Our data.

Basket credit derivatives

Basket products are those financial contracts whose payout depends on the credit event characterising a portfolio of bonds over a determinated time horizon: thus their underling is the credit quality of more *reference entities*.

The most common *basket* is that one whose *payout* depend on the temporal ranking of the credit event (*first-to-default*, *second-to-default*).

Fundamental aspects :

- ✓ Default probability: in order to describe the survival time of each defaultable reference entity we introduce a variable called *time-until-default* and we construct its probability distribution.
- ✓ Joint default: once stated as compute the default probability of each *reference entity* we study how to compute the joint default probabilities.

Starting from the *credit curves* we can determine the marginal conditional default probabilities with reference to a stated time interval

Characterize default using time-until-default

We introduce a random variable called *time-until-default* that represent the length of the survival time.

The probability distribution function of the survival time T can be specified by the following distribution function

$$F(t) = \Pr(T \leq t)$$

which gives the probability that default occurs before t.

And the corresponding probability density function:

$$f(t) = dF(t)/dt$$

In studying survival data it is useful to define the *survival function* as:

$$S(t) = 1 - F(t) = \Pr(T > t)$$

that is the probability that the credit survives at time t.

Time-until-default & hazard rate

The time-until-default could be described also by the *hazard rate* function which gives the instantaneous default probability, i.e. the default probability of the credit over the time interval [x,x+Δt] if it has survived to time x:

$$\Pr\{T \le x+\Delta t \mid T > x\} = \frac{F(x+\Delta t)-F(x)}{1-F(x)} \cong \frac{f(x)\cdot \Delta t}{1-F(x)}$$

where the *hazard rate* function is:

$$h(x) = \frac{f(x)}{1-F(x)}$$

it gives the value of the conditional probability density of T, i.e. the probability of default exactly at time x given the survival to that time.

Time-until-default & hazard rate

The survival function can be expressed in terms of the *hazard rate* as:

$$S(t) = e^{-\int_0^t h(s)ds}$$

The probability density function of survival time of a credit can also be expressed using the *hazard rate* function:

$$f(t) = h(t) \cdot S(t) = h(t) \cdot e^{-\int_0^t h(s)ds}$$

A typical assumption is that the *hazard rate* is a constant over certain period [x,x+Δt], in this case the density function is:

$$f(t) = h \cdot e^{-ht}$$

Basket products

The approach we used to pricing basket products is based on the Duffie's model so-called "reduced" form. The idea behind this approach is that the credit event can be modelled as a *Poisson* process, with intensity rate *h* depending on the length of the time interval.

The general form of the *Poisson* distribution is:

$$Poisson(x,h) = \frac{e^{-h \cdot t} \cdot (h \cdot t)^x}{x!}$$

if we consider one company we are interested only in the case $x=0$, i.e. when the company is not in default (in each other case, $x > 0$, the company is in default).

The probability that a company defaults before time *t* (the so-called "survival time") is measured by the *time-until-default* which function is Exponentially distributed:

$$F(t) = \Pr(T \leq t) = 1 - e^{-h \cdot t}$$

Pricing a first-to-default

The probability that each *reference entity* will survive until time t is:

$$S(t) = \Pr(T > t) = e^{-h \cdot t}$$

with *hazard rate h*

Given N *reference entities*, the multivariate survival joint-distribution is:

$$S(t) = e^{-H \cdot t}$$

with H = $\begin{cases} h_1 + h_2 + \ldots + h_N & \text{independent issuer} \\ h_1 + h_2 + \ldots + h_N + h_{1 \ldots N} & \text{if a } \textbf{systematic risk factor} \text{ exist} \end{cases}$

→ or also $N \cdot h + h_{1 \ldots N} = N \cdot \hat{h} - h_{1 \ldots N}$

if all the issuers are **equally** risky

Incidence of the systematic factor

At total individual *hazard rate* parity $\left(\hat{h}\right)$, the two components

h ⟶ idiosyncratic risk

$h_{1\ldots N}$ ⟶ **systematic** risk

have different influence due to the increasing of the correlation between the lender.

In particular
$$h = \hat{h}\frac{1-\rho}{1+\rho}$$

$$h_{12\ldots N} = 2\hat{h}\frac{\rho}{1+\rho}$$

In fact if
$$\rho = 0 \longrightarrow \begin{cases} h = \hat{h} \\ h_{1\ldots N} = 0 \end{cases}$$

$$\rho = 1 \longrightarrow \begin{cases} h = 0 \\ h_{1\ldots N} = \hat{h} \end{cases}$$

Pricing a first-to-default

The price of the *first-to-default option* is:

$$P = L \cdot (1 - RR) \cdot \int_0^T e^{-rt} H e^{-H \cdot t} dt$$

nominal value — loss given default — total hazard rate of the portfolio — density function

$$= L \cdot (1 - RR) \cdot \frac{H}{r + H} \left(1 - e^{-T(r+H)}\right)$$

If we simulate different correlation values and keep constant $\hat{h}_i$

$$\uparrow \rho \Rightarrow \uparrow h_{1 \ldots N} \Rightarrow \hat{h}_i Cost. \Rightarrow \downarrow H \Rightarrow \downarrow P$$

Implementation of the model

For the implementation we have considered the same *reference entities* we used with the H&W model. Using the following *risk-free* rates

years	*risk-free rate*
0	2,500%
1	2,239%
2	2,256%
3	2,539%
4	2,796%
5	3,012%

Table : *Risk-free* rates from today up to 5 years.

Source : Spot curve, Bloomberg.

and assuming a constant *hazard rate* function between each time interval (i.e. $h(t) = h_i$ with $t_{i-1} \leq t < t_i$.)
we can compute the different *hazard rates* using the following expression:

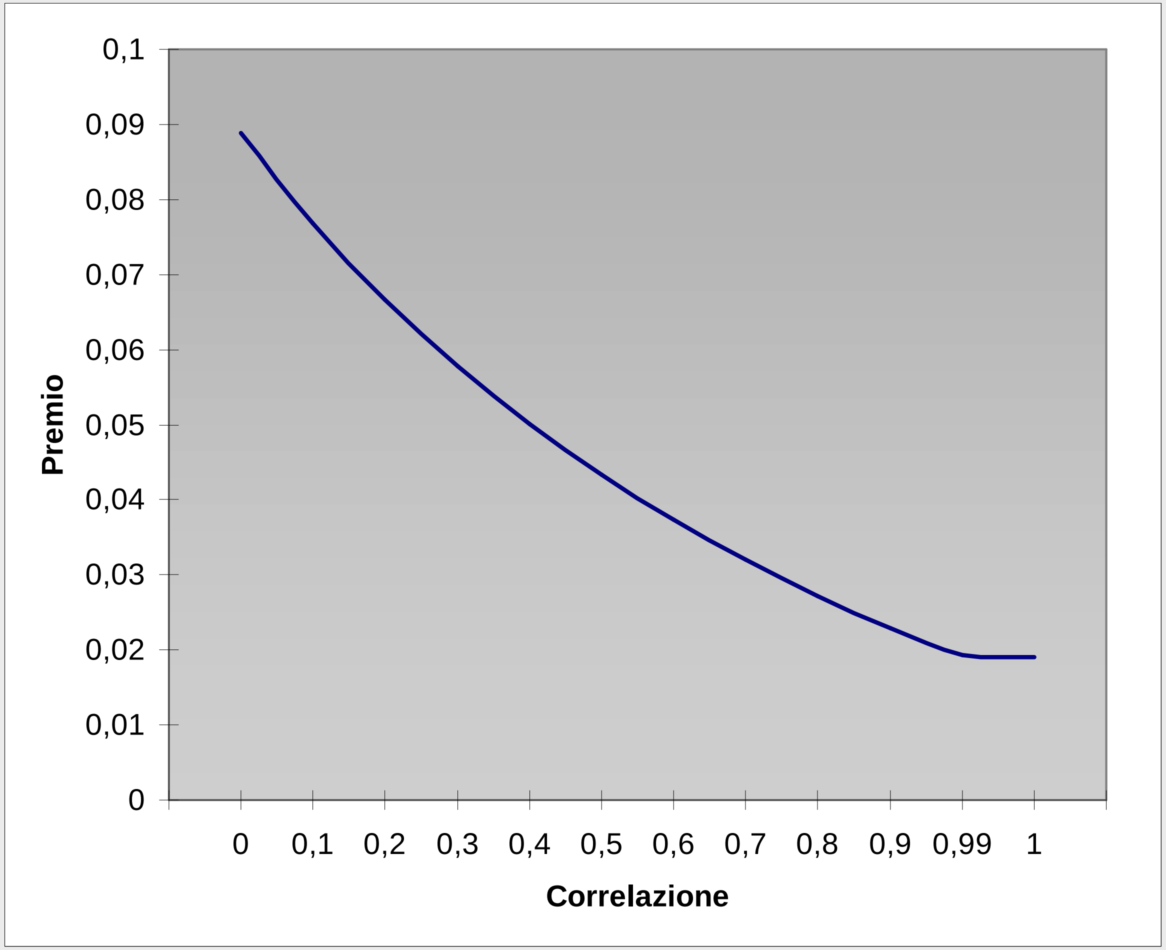
$$V(t_0) = \sum_{i=1}^{n} c_i \cdot e^{-\int_0^{ti} [r(s) + (1 - R(s)) \cdot h(s)] ds}$$

Implementation of the model

- ✓ Using the pricing formula for a basket and considering the possible correlation values existing between the *reference entities* included in the basket (we assume a nominal invested capital *L* equal to one and a *recovery rate RR = 39,46%*) we get:

Correlation	*Premio in bps.*
0	888
0,1	768
0,2	665
0,3	577
0,4	500
0,5	433
0,6	373
0,7	320
0,8	272
0,9	228
0,99	193
1	189

Price of the first-to-default



~~Diversification reduce the risk~~

Not for a *first-to-default*

Comparing the two strategies: CDS

Applying the strategy based on *credit default swap options* we get the following results:

Reference Entity	*Spread in bps*	*Price in euro (1.000)*
Carrefour S.A.	72,06928969	720.693
Dresdner Bank A.G.	86,20980168	862.098
Enel SpA	81,02908642	810.291
Unilever NV	74,46061246	744.606
Volkswagen Finance	59,20932894	592.093
Totale	372,9781192	3.729.781

That mean that starting from an initial capital of 500 millions euro, equally divided into 5 corporate bonds, we have to spend 3.729.780 euro to carry out an hedging strategy against the default of the 5 different reference entities.

Comparing the two strategies: FTD

Applying an hedging strategy based on a *first-to-default option* we can see different prices that depend on the correlation level between the *names* included in the basket :

Correlation	*Price in bps.*	*Price in euro (1.000)*
0	888,5789389	8885,789
0,1	768,3775278	7683,775
0,2	665,9541116	6659,541
0,3	577,6528318	5776,528
0,4	500,7517336	5007,517
0,5	433,1836945	4331,837
0,6	373,3513632	3733,514
0,7	320,0012821	3200,013
0,8	272,1363056	2721,363
0,9	228,9534193	2289,534
0,99	193,5490871	1935,491
1	189,7987829	1897,988

Conclusions

- The model reacts correctly to the variation of the correlation value
  - ✓ negative correlation → spread tend to high level
- However it requires some arbitrary assumption:
  - ✓ Identical credit quality for the lenders;
  - ✓ only one systematic factor.

OBJECTIVE of my research

- Improve the *pricing* model using:
  - ✓ stochastic individual *hazard rates;*
  - ✓ correlating survival times using Copula functions.