RISK-ADJUSTED

PERFORMANCE ATTRIBUTION

METHODOLOGIES

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OUTLINE:

- performance measures of managed funds
- how to adjust for risk
- the axiomatics of performance attribution
- multi-period performance attribution
- fixed-income performance attribution

(RATE OF) RETURN

- ex–ante

- ex–post

$$\begin{aligned} V(0) &\Longrightarrow V(T) \\ R[0,T] &= \frac{V(T) - V(0)}{V(0)} \end{aligned}$$

This is the *periodic* rate of return. How to annualize?

  - simple rule

$$r = \frac{1}{T} R[0,T]$$

  - compound rule

$$r = (1 + R[0,T])^{1/T} - 1$$

HOW TO MANAGE INTERMEDIATE CASH-FLOWS

- time–weighted rate of return

$$
\begin{aligned}
R[T_{i-1}, T_i] &= \frac{V(T_i)}{V(T_{i-1}) + F(T_i)} - 1 \\
1 + R[0, T] &= \prod_i (1 + R[T_{i-1}, T_i])
\end{aligned}
$$

(enhances the management skills) / consistent with compound rule!

- money–weighted rate of return

$$
\begin{aligned}
W(T) &= V(0) + \sum \frac{T - T_i}{T} F(T_i) \\
R[0, T] &= \frac{V(T)}{W(T)} - 1
\end{aligned}
$$

rough approximation / consistent with simple rule

WHAT IS "RISK"???

when (and the extent to which) the outcome of an action differs negatively from what expected (hoped, forecasted,...)

[Ref.: Elton, 2000]

Examples *(internal measures)*:

- standard deviation:

$$\sigma = \left(\sum p_i \left(r_i - \mathbb{E}(r)\right)^2\right)^{1/2}$$

- variance

$$\sigma^2 = \sum p_i \left(r_i - \mathbb{E}(r)\right)^2$$

- semivariance

$$s = \sum p_i \left(r_i - \mathbb{E}(r)\right)^+$$

Examples *(external measures, wrt a target or benchmark)*

- downside deviation

$$DD = \left[\frac{1}{T}\sum\left(b_t - r_t\right)^+\right]^{1/2}$$

- tracking error volatility

$$TEV = \frac{1}{T}\sum\left(b_t - r_t\right)^2$$

- shortfall probability

$$SFP = \frac{1}{T}\left[\#\left(b_t - r_t\right)^+\right]$$

RISK–ADJUSTED PERFORMANCE MEASURES

- Sharpe index

$$SI = \frac{\mu_P - r_f}{\sigma_P}$$

- Modigliani

$$RAPM = r_f + \left(\mu_P - r_f\right)\left[\frac{\sigma_B}{\sigma_P}\right]$$

- Sortino

$$Sort = \frac{\mu_P - \mu_B}{DD}$$

- Information Ratio

$$IR = \frac{\mu_{TE}}{TEV}$$

- Jensen's alpha

$$\alpha = \mu_P - \left[r_f + \beta_P \left(r_M - r_f\right)\right]$$

- Treynor

$$TI = \frac{\mu_P - r_f}{\beta_P}$$

what is Performance Attribution?

" ... the methodology whereby the excess return is decomposed into components, each referring to a particular aspect of the strategic process of portfolio management."

BENCHMARK

$\Downarrow \qquad \Uparrow$

ASSET ALLOCATION

$\Downarrow \qquad \Uparrow$

MARKET TIMING

$\Downarrow \qquad \Uparrow$

STOCK SELECTION

(top-down vs. bottom-up)

Arithmetic Excess Return:

$$\begin{aligned} AER &= R - B = \sum w_i R_i - \sum v_i B_i = \\ &= \underbrace{\sum (w_i - v_i) R_i}_{\text{Stock Selection}} + \underbrace{\sum v_i (R_i - B_i)}_{\text{Market Timing}} \end{aligned}$$

Geometric Excess Return

$$\begin{aligned} GER &= \frac{1+R}{1+B} = \prod (1+R_i)^{w_i} : \prod (1+B_i)^{v_i} \\ &= \underbrace{\prod (1+R_i)^{w_i - v_i}}_{\text{Stock Selection}} \times \underbrace{\prod \left(\frac{1+R_i}{1+B_i} \right)^{v_i}}_{\text{Market Timing}} \end{aligned}$$

MULTIPERIOD COMPOUNDING

Geometric compounding is straightforward:

$$GER_{1|T} = \frac{1 + R_{1|T}}{1 + B_{1|T}} = \prod_t \frac{1 + R_t}{1 + B_t}$$

Arithmetic compounding:

$$\begin{aligned} AER_{1|T} &= R_{1|T} - B_{1|T} \\ &\neq \sum_t (R_t - B_t) \end{aligned}$$

Equality can be recovered *only* if the one-period returns are evaluated on the basis of the initial endowment:

$$R_t = \frac{V_t - V_{t-1}}{V_0}$$

MAIN SOURCES OF RISK

(=POTENTIAL EXCESS RETURN)

- Market risk (changes in the Term Structure)

- Default risk

- Currency risk

$$ex - post \text{ rate of return} = \frac{V_1 - V_0}{V_0}$$

(time-weighted vs. money-weighted)

(*tel-quel* quotes)

If market conditions are stable, the value will follow a pattern such as to yield the same rate of return as established at time 0. Hence, part of the price variation is *expected.*

We are concerned with the *unexpected* part (if any), the only one for which active management deserves a reward.

Usually attributed to duration.

Quoting van Breukelen:

$$LR = C + \mathcal{D} \cdot (-\Delta y)$$

Reasons why duration is NOT a correct metric:

- the initial TS is flat $\Longrightarrow$ all YTM are equal to the common value of the spot rate.

- parallel shifts $\Longrightarrow$ all YTM remain equal.

Only under these assumptions duration is a correct measure of sensitivity to changes in "market rate(s)". Neither of these is satisfied under actual situations:

- the initial TS is usually upward-sloping and concave;

- the most widely recognized movements are
  - shifts
  - twists
  - butterflies (humps)

"Isolate" the effect of every single component of the TS.

Hypotheses:

1. the term structure of interest rates is the only risk factor of the market;

2. all bonds are correctly priced according to the term structure
$$P_0 = \sum_{h=1}^{n} \left[\frac{c_h}{\prod_{j=1}^{h} \left(1 + r_j\right)} \right]$$

3. the rational expectations theory holds (namely, one-period forward rates are expected to become the spot rates next period, and so on).

$$P_1 = \underbrace{\sum_{h=2}^{\nu-1}\left[\frac{c_h}{\prod_{j=2}^{h}\left(1+r_j\right)}\right]}_{\text{independent from } r_\nu} + $$

$$+\frac{1}{(1+r_\nu)} \underbrace{\sum_{h=\nu}^{n}\left[\frac{c_h}{\prod_{j=2,\neq\nu}^{h}\left(1+r_j\right)}\right]}_{\text{independent from } r_\nu}$$

Taking the derivative wrt a specific rate:

$$\frac{\partial P_1}{\partial r_\nu} = \frac{-1}{(1+r_\nu)} \sum_{h=\nu}^{n}\left[\frac{c_h}{\prod_{j=2}^{h}\left(1+r_j\right)}\right]$$

Aggregating the changes in all the TS rates:

$$\Delta P_1 \simeq \sum_{\nu=1}^{n} \frac{\partial P_1}{\partial r_\nu}\Delta r_\nu =$$

$$= \sum_{\nu=1}^{n}\left\{-\frac{1}{(1+r_\nu)}\sum_{h=\nu}^{n}\left[\frac{c_h}{\prod_{j=2}^{h}\left(1+r_j\right)}\right]\right\}\Delta r_\nu$$

This formula can accommodate virtally every reshape of the TS: for instance

- shift:

$$\Delta r_\nu = \alpha$$

- twist:

$$\Delta r_\nu = \square \cdot (\nu - \nu^*)$$

- butterfly:

$$\Delta r_\nu = \eta + \gamma \cdot (\nu - \nu^*)^2$$

- etc.

These "rules" can be fitted into the general formula to exhibit more compact expressions of the total price variation.

Main references:

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G. van Breukelen, (2000), "Fixed-Income Attribution", *Journal of Performance Measurement*