RISK-ADJUSTED PERFORMANCE ATTRIBUTION METHODOLOGIES

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OUTLINE:

• performance measures of managed funds

• how to adjust for risk

• the axiomatics of performance attribution

• multi-period performance attribution

• fixed-income performance attribution
(RATE OF) RETURN

• ex–ante

• ex–post

\[ V(0) \implies V(T) \]
\[ R[0,T] = \frac{V(T) - V(0)}{V(0)} \]

This is the \textit{periodic} rate of return. How to annualize?

– simple rule

\[ r = \frac{1}{T} R[0,T] \]

– compound rule

\[ r = (1 + R[0,T])^{1/T} - 1 \]
HOW TO MANAGE INTERMEDIATE CASH-FLOWS

• time–weighted rate of return

\[ R[T_{i-1}, T_i] = \frac{V(T_i)}{V(T_{i-1}) + F(T_i)} - 1 \]

\[ 1 + R[0, T] = \prod_{i} (1 + R[T_{i-1}, T_i]) \]

(enhances the management skills) / consistent with compound rule!

• money–weighted rate of return

\[ W(T) = V(0) + \sum \frac{T - T_i}{T} F(T_i) \]

\[ R[0, T] = \frac{V(T)}{W(T)} - 1 \]

rough approximation / consistent with simple rule
WHAT IS ”RISK”???

when (and the extent to which) the outcome of an action differs negatively from what expected (hoped, forecasted,...)

[Ref.: Elton, 2000]

Examples (internal measures):

- **standard deviation:**
  \[
  \sigma = \left( \sum p_i (r_i - \mathbb{E}(r))^2 \right)^{1/2}
  \]

- **variance**
  \[
  \sigma^2 = \sum p_i (r_i - \mathbb{E}(r))^2
  \]

- **semivariance**
  \[
  s = \sum p_i (r_i - \mathbb{E}(r))^+
  \]
Examples (external measures, wrt a target or benchmark)

- downside deviation
  \[ DD = \frac{1}{T} \sum (b_t - r_t)^{+} \]^{1/2}

- tracking error volatility
  \[ TEV = \frac{1}{T} \sum (b_t - r_t)^2 \]

- shortfall probability
  \[ SFP = \frac{1}{T} \left[ \# (b_t - r_t)^{+} \right] \]
RISK–ADJUSTED PERFORMANCE MEASURES

- Sharpe index

\[ SI = \frac{\mu_P - r_f}{\sigma_P} \]

- Modigliani

\[ RAPM = r_f + (\mu_P - r_f) \left[ \frac{\sigma_B}{\sigma_P} \right] \]

- Sortino

\[ Sort = \frac{\mu_P - \mu_B}{DD} \]

- Information Ratio

\[ IR = \frac{\mu_{TE}}{TEV} \]
• Jensen’s alpha

\[ \alpha = \mu_P - \left[ r_f + \beta_P \left( r_M - r_f \right) \right] \]

• Treynor

\[ TI = \frac{\mu_P - r_f}{\beta_P} \]
what is Performance Attribution?

"... the methodology whereby the excess return is decomposed into components, each referring to a particular aspect of the strategic process of portfolio management."

BENCHMARK

\[ \downarrow \quad \uparrow \]

ASSET ALLOCATION

\[ \downarrow \quad \uparrow \]

MARKET TIMING

\[ \downarrow \quad \uparrow \]

STOCK SELECTION

(top-down vs. bottom-up)
Arithmetic Excess Return:

\[
AER = R - B = \sum w_i R_i - \sum v_i B_i = \\
= \sum (w_i - v_i) R_i + \sum v_i (R_i - B_i)
\]

\text{Stock Selection} \quad \text{Market Timing}

Geometric Excess Return

\[
GER = \frac{1 + R}{1 + B} = \prod (1 + R_i)^{w_i} : \prod (1 + B_i)^{v_i}
\]

\[
= \prod (1 + R_i)^{w_i-v_i} \times \prod \left(\frac{1 + R_i}{1 + B_i}\right)^{v_i}
\]

\text{Stock Selection} \quad \text{Market Timing}
MULTIPERIOD COMPOUNDING

Geometric compounding is straightforward:

\[ GER_{1|T} = \frac{1 + R_{1|T}}{1 + B_{1|T}} = \prod_t \frac{1 + R_t}{1 + B_t} \]

Arithmetic compounding:

\[ AER_{1|T} = R_{1|T} - B_{1|T} \neq \sum_t (R_t - B_t) \]

Equality can be recovered only if the one-period returns are evaluated on the basis of the initial endowment:

\[ R_t = \frac{V_t - V_{t-1}}{V_0} \]
MAIN SOURCES OF RISK

(=POTENTIAL EXCESS RETURN)

• Market risk (changes in the Term Structure)

• Default risk

• Currency risk
If market conditions are stable, the value will follow a pattern such as to yield the same rate of return as established at time 0. Hence, part of the price variation is expected.

We are concerned with the unexpected part (if any), the only one for which active management deserves a reward.

Usually attributed to duration.

Quoting van Breukelen:

\[ LR = C + D \cdot (-\Delta y) \]
Reasons why duration is NOT a correct metric:

- the initial TS is flat $\implies$ all YTM are equal to the common value of the spot rate.

- parallel shifts $\implies$ all YTM remain equal.

Only under these assumptions duration is a correct measure of sensitivity to changes in "market rate(s)". Neither of these is satisfied under actual situations:

- the initial TS is usually upward-sloping and concave;

- the most widely recognized movements are
  - shifts
  - twists
  - butterflies (humps)
"Isolate" the effect of every single component of the TS.

Hypotheses:

1. the term structure of interest rates is the only risk factor of the market;

2. all bonds are correctly priced according to the term structure

\[
P_0 = \sum_{h=1}^{n} \left[ \frac{c_h}{\prod_{j=1}^{h} (1 + r_j)} \right]
\]

3. the rational expectations theory holds (namely, one-period forward rates are expected to become the spot rates next period, and so on).
\[ P_1 = \sum_{h=2}^{\nu-1} \left[ \frac{c_h}{\prod_{j=2}^{h} (1 + r_j)} \right] + \]

\[ \left. \right. + \frac{1}{(1 + r_\nu)} \sum_{h=\nu}^{n} \left[ \frac{c_h}{\prod_{j=2, j \neq \nu}^{h} (1 + r_j)} \right] \]

independent from \( r_\nu \)

Taking the derivative wrt a specific rate:

\[ \frac{\partial P_1}{\partial r_\nu} = \frac{-1}{(1 + r_\nu)} \sum_{h=\nu}^{n} \left[ \frac{c_h}{\prod_{j=2, j \neq \nu}^{h} (1 + r_j)} \right] \]

Aggregating the changes in all the TS rates:

\[ \Delta P_1 \approx \sum_{\nu=1}^{n} \frac{\partial P_1}{\partial r_\nu} \Delta r_\nu = \]

\[ = \sum_{\nu=1}^{n} \left\{ -\frac{1}{(1 + r_\nu)} \sum_{h=\nu}^{n} \left[ \frac{c_h}{\prod_{j=2, j \neq \nu}^{h} (1 + r_j)} \right] \right\} \Delta r_\nu \]
This formula can accommodate virtually every reshape of the TS: for instance

- **shift:**
  \[ \Delta r_\nu = \alpha \]

- **twist:**
  \[ \Delta r_\nu = \beta (\nu - \nu^*) \]

- **butterfly:**
  \[ \Delta r_\nu = \eta + \gamma (\nu - \nu^*)^2 \]

- etc.

These "rules" can be fitted into the general formula to exhibit more compact expressions of the total price variation.
Main references:


E. M. Ankrim, (1990), "Risk-Adjusted Performance Attribution", Russell Research Commentaries, December