GOALS AND CONTENTS OF THE COURSE

The didactical goal is to introduce the basis for the numerical solution of the linear systems, the numerical approximation of ordinary differential equations, and finite difference methods for the discretization of partial differential equations. Moreover, we will introduce the variational formulation of some boundary value problems together with the finite element method for their numerical approximation. The course is characterized by a constant synergy between modeling, theoretical aspects and numerical simulation.

TOPICS

First Part – Review of numerical linear algebra

1. **Solution of linear systems**

Gaussian elimination method, LU factorization, iterative methods, coniugate gradient, GMRes, preconditioning.

Second Part – Differential modeling and finite difference approximation

2. ***Ordinary differential equations***

Finite difference formulae to approximate derivatives. Numerical approximation of ordinary differential equations, convergence, absolute stability.

3. ***First-order conservation laws***

Approximation with finite differences. Convergence, consistency, zero-stability and absolute stability. Forward Euler-centered scheme. Upwind, Lax-Friedrichs and Lax-Wendroff schemes. Analysis of the schemes, CFL condition and its meaning. Backward Euler-centered scheme.

4. ***Laplace-Poisson equation***

Discretization with finite differences of a one-dimensional elliptic problem. Imposition of the Dirichlet and Neumann boundary conditions. Algebraic formulation e matrix properties. Diffusion-convection and diffusion-reaction problems.

5. **Heat equation**

Discretization of the heat equation with finite differences. Implicit and explicit time marching schemes, the theta-method, stability analysis.

6. ***Wave equation***

Numerical Methods: Discretization of the wave equation with finite difference explicit and implicit schemes. Leapfrog and Newmark schemes. Stability properties.

Third Part – Variational formulations and discretizations via finite element method.

7. ***Weak formulation of stationary problems***

Introduction to the Galerkin method for a one-dimensional elliptic problem. Consistency, stability and convergence. Cea' Lemma. The finite elements method. Linear and quadratic finite elements. Definition of Lagrangian basis functions, of composite interpolation and error estimates. Approximation of the diffusion-convection-reaction problem: comparison with the

finite difference case and stability analysis. Stabilization with the upwind strategy and the mass lumping technique.

REFERENCE BOOKS:

Quarteroni, Modellistica Numerica per Problemi Differenziali, Springer Italia UNITEXT, 5a ed., 2012, ISBN: 978-88-470-2747-3.

Quarteroni, Saleri, Gervasio, Calcolo Scientifico, Springer Italia, 5a ed., 2012, ISBN:978-88-470-2744-2.